

# Formula renaming

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We extend the machinery from propositional to first-order logic:

$$\nu(\forall x \phi) = \nu(\exists x \phi) = \nu(\phi) \text{ and } \bar{\nu}(\forall x \phi) = \bar{\nu}(\exists x \phi) = \bar{\nu}(\phi).$$

Introduce top-down fresh predicates for beneficial subformulas:

$$\psi[\phi]_p \Rightarrow_{\text{OCNF}} \psi[P(x_1, \dots, x_n)]_p \wedge \text{def}(\psi, p, P)$$

where  $\{x_1, \dots, x_n\}$  are the free variables in  $\phi$ ,  $P/n$  is a predicate new to  $\psi[\phi]_p$ ,  $\nu(\psi[\phi]_p) > \nu(\psi[P]_p \wedge \text{def}(\psi, p, P))$ , and  $\text{def}(\psi, p, P)$  is defined polarity dependent analogous to the propositional case:

$$\text{def}(\psi, p, P) := \forall x_1, \dots, x_n [\psi|_p \circ P(x_1, \dots, x_n)]$$

where  $\circ \in \{\rightarrow, \leftrightarrow, \leftarrow\}$ .