2.10 Superposition Versus CDCL

We will establish a relationship between Superposition and CDCL operating on a clause set N:

Superposition: Is based on an ordering $\prec$. It computes a model assumption $N_{\mathcal{I}}$. Either $N_{\mathcal{I}}$ is a model, N contains the empty clause, or there is an inference on the minimal false clause with respect to $\prec$.

CDCL: Is based on a variable selection heuristic. It computes a model assumption via decision variables and propagation. Either this assumption is a model of N, N contains the empty clause, or there is a backjump clause that is learned.

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Proposition 2.20:

Let $(L_1 + L_2 + \ldots + L_k; N)$ be a CDCL with eager propagation state. Some of the L_i may be decision literals and the corresponding propositional variables are $P_1, \ldots, P_k$. Furthermore, let us assume that $L_1 + \ldots + L_{k-1}$ is a partial valuation that does not falsify any clause in N whereas $L_1 + L_2 + \ldots + L_k$ falsifies some clause $C \vee \overline{L_k} \in N$. Then

(a) L_k is a propagated literal.

(b) The resolvent between $C \vee \overline{L_k}$ and the clause propagating L_k is a superposition inference and the conclusion is not redundant with respect to the ordering $P_1 \prec P_2 \ldots \prec P_k$.

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Proposition 2.21:
The 1UIP backjump clause is not redundant.

Proof:
By Proposition 2.20 a one resolution step 1UIP backjump clause has this property. The argument in the proof of Proposition 2.20 can be repeated until we reach the first decision literal L_m by resolving away $L_k, L_{k-1}, \ldots, L_{m+1}$. $\square$

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Proposition 2.22:
Let $(L_1 + L_2 + \ldots + L_k; N)$ be a CDCL with eager propagation state. We assume that all decision literals among the L_i are negative and let the corresponding propositional variables be $P_1, \ldots, P_k$. Furthermore, let us assume that $L_1 + \ldots + L_k$ is a partial valuation that does not falsify any clause in N. Then $N_{\mathcal{I}}^{\prec P_{k+1}} = \{P_1, \ldots, P_k\} \cap \{L_1, \ldots, L_k\}$ with ordering $P_1 \prec P_2 \ldots \prec P_{k+1}$.