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Tutorials for "Automated Reasoning"
Exercise sheet 14

Exercise 14.1: (*4 P*)
Use superposition to show that the following set of (implicitly universally quantified) clauses is not satisfiable:

$$f(a,x) \approx x$$
$$x \approx a \vee x \approx g(a)$$
$$x \not\approx g(x)$$
$$f(a,g(a)) \approx g(b)$$
$$b \not\approx a$$

Use the LPO with precedence $f > g > a > b$. Compute only inferences that are required according to the ordering restrictions of the superposition calculus.

Exercise 14.2: (*3 P*)
Let $\succ$ be the LPO with the precedence $a > b > c > g > f$. Order the following ground equational clauses according to the clause ordering $\succ_C$ defined in the lecture.

$$f(a,c) \not\approx g(f(c,c)) \vee a \approx g(c)$$
$$a \not\approx b \vee f(f(a,a),f(a,b)) \not\approx f(g(a),g(b))$$
$$g(a) \approx g(b) \vee g(f(c,a)) \approx f(a,c)$$
$$a \approx b \vee g(g(c)) \not\approx f(c,b)$$
$$g(c) \approx f(a,b)$$

Exercise 14.3: (*2 P*)
Let $\succ$ be a reduction ordering that is total on ground terms and E be a set of equations. Suppose that equations in E can be oriented (i.e. for every $t \approx s \in E$ exactly one of $t \succ s$ or

$s \succ t$ holds). Show that the set of semi-critical pairs of E with respect to $\succ$ is the same as the set of critical pairs of rewrite system R where $R = \{s \to t \mid s \succ t \wedge s \approx t \in E\}$.

Exercise 14.4: (*3 P*)
Prove Theorem 6.4.

Exercise 14.5: (*2 Bonus Points*)
Is there any reduction ordering that is total on $T_\Sigma(X)$? Give a proof or counterexample.

Submit your solution in lecture hall 001 during the lecture **on July 23**. Please write your name and the date of your tutorial group on your solution.

Note: Joint solutions are not permitted (work in groups is encouraged).