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July 2, 2012

Tutorials for “Automated Reasoning”
Exercise sheet 12

Exercise 12.1: (2 P)

Prove using the LPO that the following rewrite system is terminating.

$$\begin{aligned}g(a, y) &\rightarrow f(y) \\g(f(x), a) &\rightarrow g(x, f(a)) \\g(f(x), f(y)) &\rightarrow g(x, g(f(x), y))\end{aligned}$$

Exercise 12.2: (2 P)

Prove using the KBO that the following rewrite system is terminating.

$$\begin{aligned}g(f(x), g(y, z)) &\rightarrow g(x, g(f(f(y)), z)) \\g(f(x), g(y, g(z, w))) &\rightarrow g(x, g(z, g(y, w)))\end{aligned}$$

Exercise 12.3: (4 P)

Compute all critical pairs and decide which of the following rewrite systems are locally confluent:

- a) $f(g(f(x))) \rightarrow x, f(g(x)) \rightarrow g(f(x))$
- b) $h(x, x) \rightarrow a, h(x, f(x)) \rightarrow b$
- c) $f(f(x, y), z) \rightarrow f(x, f(y, z)), f(x, a) \rightarrow x$
- d) $f(f(x, y), z) \rightarrow f(x, f(y, z)), f(a, x) \rightarrow x$

Exercise 12.4: (3 P)

Let E be a set of equations, let $\sigma : X \rightarrow T_{\Sigma}(X)$ be a substitution. Prove that $E \vdash t \approx t'$

implies $E \vdash t\sigma \approx t'\sigma$ for all terms t, t' over Σ . Does the reverse implication also hold? Prove or show a counterexample.

Exercise 12.5: (2 Bonus Points)

A term rewriting system R is called *right-reduced* if for all $(l \rightarrow r) \in R$, r is R -irreducible. Prove or disprove: a right-reduced rewrite system consisting of finitely many rules $l \rightarrow r$ such that r is a ground term is terminating.

Submit your solution in lecture hall 001 during the lecture **on July 9**. Please write your name and the date of your tutorial group on your solution.

Note: Joint solutions are not permitted (work in groups is encouraged).