UNIVERSITÄT DES SAARLANDES
FR 6.2 – Informatik
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Lecture "Automated Reasoning" (Summer Term 2012)

Midterm Examination

Name: ....

Student Number: ....

Some notes:

- Things to do at the beginning:

  Put your student card and identity card (or passport) on the table.
  Switch off mobile phones.
  Whenever you use a new sheet of paper (including scratch paper), first write your name and student number on it.

- Things to do at the end:

  Mark every problem that you have solved in the table below.
  Stay at your seat and wait until a supervisor staples and takes your examination text.
  Note: Sheets that are accidentally taken out of the lecture room are invalid.

Sign here: **Good luck!**

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Problem	1	2a	2b	2c	3	4	5	6a	6b	7	Σ
Answered?											
Points											

Problem 1 *(Superposition)* (8 points)

Show unsatisfiability of the below clauses via the superposition calculus based on the atom ordering $P2 \succ P1 \succ Q \succ R$:

(1) $\neg P1 \vee Q \vee R$

(2) $\neg P2 \vee P1 \vee R$

(3) $P2 \vee P1 \vee R$

(4) $\neg R \vee Q$

(5) $\neg P1 \vee R$

(6) $\neg P1 \vee \neg Q$

(7) $\neg R \vee P1$

Problem 2 *(Superposition Model Building)* (4 + 1 + 2 = 7 points)

Consider again the clause set N of Problem 1, containing the below clauses, but now with *different* atom ordering $P1 \succ P2 \succ Q \succ R$.

$$N = \{\neg P1 \vee Q \vee R, \neg P2 \vee P1 \vee R, P2 \vee P1 \vee R, \neg R \vee Q, \neg P1 \vee R, \neg P1 \vee \neg Q, \neg R \vee P1\}$$

(a) Compute $N_{\mathcal{I}}$.

(b) Determine the minimal false clause and its productive counterpart, producing the atom of the maximal negative literal in the false clause.

(c) Compute the superposition inference out of (b), add it to N resulting in N' and compute $N'_{\mathcal{I}}$.

Problem 3 *(CDCL)* (6 points)

Check via the rule-based CDCL calculus ($\Rightarrow_{\mathrm{DPLL}}$ + learning rule) whether the following set of clauses is satisfiable or not. Learn backjump clauses.

$N = \{\neg P1 \vee Q \vee P3, \neg P2 \vee P1 \vee R, P2 \vee P1 \vee R, \neg R \vee Q, \neg P3 \vee R, \neg P1 \vee \neg Q, \neg P3 \vee P1\}$

Problem 4 *(CNF)* (6 points)

Transform the formula

$$\neg((P \vee Q) \leftrightarrow (P \rightarrow (Q \wedge \top)))$$

into CNF using $\Rightarrow_{\text{OCNF}}$ without applying Step 2 (introduction of new propoitional variables) of the procedure.

Problem 5 *(Validity)* (4 points)

Let ϕ, ψ, χ be propositional formulas. Prove that the formula

$$(\psi \to \chi) \to [(\phi \lor \psi) \to (\phi \lor \chi)]$$

is valid.

Problem 6 *(Modularity)* (2 + 2 = 4 points)

Prove or give a counter example for the following statements.

(a) If for the formula $\phi_1 \vee \phi_2$ there is a valuation $\mathcal{A}$ with $\mathcal{A}(\phi_1) = 1$ then $\phi_1 \vee \phi_2$ is satisfiable.

(b) If for the formula $\phi_1 \wedge \phi_2$ there are two valuations $\mathcal{A}_1$, $\mathcal{A}_2$ with $\mathcal{A}_1(\phi_1) = 1$ and $\mathcal{A}_2(\phi_2) = 1$ then $\phi_1 \wedge \phi_2$ is satisfiable.

Problem 7 *(Clause Set Reduction)* (1 + 4 = 5 points)

Consider a finite set N of propositional clauses and a propositional atom P occurring in N. Assume N contains no tautologies, i.e., no clauses of the form $Q \lor \neg Q \lor C$ for some propositional variable Q. Further, we assume that any resolvent on P with clauses from N is a tautology. Consider the partitioning

$$N = N' \uplus \{P \lor C_i \mid (P \lor C_i) \in N\} \uplus \{\neg P \lor D_j \mid (\neg P \lor D_j) \in N\}$$

Prove N is satisfiable iff N' is satisfiable. Note that N' does not contain P anymore.