UNIVERSITÄT DES SAARLANDES
FR 6.2 – Informatik
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Lecture "Automated Reasoning"
(Summer Term 2012)

Final Examination

Name: ....

Student Number: ....

Some notes:

- Things to do at the beginning:

  Put your student card and identity card (or passport) on the table.
  Switch off mobile phones.
  Whenever you use a new sheet of paper (including scratch paper), first write your name and student number on it.

- Things to do at the end:

  Mark every problem that you have solved in the table below.
  Stay at your seat and wait until a supervisor staples and takes your examination text.
  Note: Sheets that are accidentally taken out of the lecture room are invalid.

Sign here: **Good luck!**

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Problem	1	2a	2b	2c	3	4	5	6	7	Σ
Answered?										
Points										

Problem 1 *(DPLL)* (6 points)

Check via the rule-based CDCL calculus ($\Rightarrow_{\mathrm{DPLL}}$ + learning rule) whether the following clause set is satisfiable or not. Learn backjump clauses.

$$\{\neg P1 \vee Q \vee R, \neg P2 \vee P1 \vee R, P2 \vee P1 \vee R, \neg R \vee Q, \neg P1 \vee R, \neg P1 \vee \neg Q, \neg R \vee P1\}$$

Problem 2 *(Superposition Model Building)* (8 points)

Consider the following clause set N with respect to an LPO where $g \succ f \succ b \succ a$.

$$N = \{f(a,b) \approx b, b \approx a \lor b \approx g(a), b \not\approx g(b), f(a, g(a)) \approx g(b), b \not\approx a\}$$

(a) Compute R_∞.

(b) Determine the minimal false clause.

(c) Compute the superposition inference out of (b), add it to the clause set N compute the new respective R_∞.

Problem 3 *(Unification)* (6 points)

Solve the below unification problem using $\Rightarrow_{PU}$ and present the eventual unifier where x, y, z, u, v are all variables.

$$E = \{f(x, g(y)) \doteq f(f(z, y), u), g(u) \doteq v\}$$

Problem 4 *(CNF)* (6 points)

Apply the CNF algorithm of Section 3.6 from the lecture plus the eventual transformation to clauses to the below first-order formula. There is no beneficial subformula to rename.

$$\forall x\, \exists y\, (f(y, y) \approx y \vee (g(x) \approx x \wedge \forall y\, f(y, x) \approx g(y)))$$

Problem 5 *(Completion)* (8 points)

Apply completion ($\Rightarrow_{\text{KBC}}$) to the following set of equations with respect to a KBO where all signature symbols (and variables) have weight 1 and $f \succ g \succ b \succ a$.

$$N = \{f(g(x), x) \doteq\approx f(x, x), f(g(a), b) \doteq\approx f(b, a), g(g(x)) \doteq\approx g(x)\}$$

Problem 6 *(Saturation)* (6 points)

Determine an ordering and a selection function such that for the below clause set no superposition inference is possible. As usual, a, b are constants and x, y are variables. Show the maximal/selected literals and argue why there is no inference.

$$N = \{g(a) \approx b, f(g(x), x) \approx x \vee f(x, g(x)) \approx x, f(a, a) \not\approx a \vee f(x, y) \approx g(y)\}$$

Problem 7 *(Superposition Termination)* (7 points)

Let N be a finite set of first-order clauses all having the form $D \vee f(t_1, t_2) \approx a$ where D might be empty or contains only disequations. The function f does not occur in t_1, t_2. As usual t_1, t_2 are terms and a is a constant.

Furthermore, with respect to some reduction ordering $\succ$, assume that for all clauses in N the equation $f(t_1, t_2) \approx a$ is strictly maximal, $f(t_1, t_2) \succ a$, and $\text{vars}(D) \subseteq \text{vars}(f(t_1, t_2))$. Finally, for any variable x occurring in D and $f(t_1, t_2)$ for some clause $D \vee f(t_1, t_2) \approx a$, the depth of x in a disequation in D is smaller or equal than the depth of x in $f(t_1, t_2)$.

Prove that for *any* ground disequation $s \not\approx t$ superposition terminates on $N \cup \{s \not\approx t\}$.