3.16 Other Inference Systems

- Instantiation-based methods
  Resolution-based instance generation
  Disconnection calculus
  ...
- Natural deduction
- Sequent calculus/Gentzen calculus
- Hilbert calculus

Instantiation-Based Methods for FOL

Idea:

Overlaps of complementary literals produce instantiations (as in resolution);

However, contrary to resolution, clauses are not recombined.

Instead: treat remaining variables as constant and use efficient propositional proof methods, such as DPLL.

There are both saturation-based variants, such as partial instantiation [Hooker et al.] or resolution-based instance generation (Inst-Gen) [Ganzinger and Korovin], and tableau-style variants, such as the disconnection calculus [Billon; Letz and Stenz].

Natural Deduction

Natural deduction (Prawitz):

Models the concept of proofs from assumptions as humans do it (cf. Fitting or Huth/Ryan).

Sequent Calculus

Sequent calculus (Gentzen):

Assumptions internalized into the data structure of sequents

$$F_1, \dots, F_m \vdash G_1, \dots, G_k$$

meaning

$$F_1 \wedge \cdots \wedge F_m \rightarrow G_1 \vee \cdots \vee G_k$$

A kind of mixture between natural deduction and semantic tableaux.

Inferences rules, e.g.:

$$\frac{\Gamma \vdash \Delta}{\Gamma, F \vdash \Delta} \ (WL) \qquad \frac{\Gamma, F \vdash \Delta \qquad \Sigma, G \vdash \Pi}{\Gamma, \Sigma, F \vee G \vdash \Delta, \Pi} \ (\vee L)$$

$$\frac{\Gamma \vdash \Delta}{\Gamma \vdash F, \Delta} \ (WR) \qquad \frac{\Gamma \vdash F, \Delta \qquad \Sigma \vdash G, \Pi}{\Gamma, \Sigma \vdash F \wedge G, \Delta, \Pi} \ (\wedge R)$$

Perfect symmetry between the handling of assumptions and their consequences.

Can be used both backwards and forwards.

Hilbert Calculus

Hilbert calculus:

Direct proof method (proves a theorem from axioms, rather than refuting its negation)

Axiom schemes, e. g.,

$$F \rightarrow (G \rightarrow F)$$
$$(F \rightarrow (G \rightarrow H)) \rightarrow ((F \rightarrow G) \rightarrow (F \rightarrow H))$$

plus Modus ponens:

$$\frac{F \qquad F \rightarrow G}{G}$$

Unsuitable for finding or reading proofs, but sometimes used for *specifying* (e.g. modal) logics.