2.7 Example: Sudoku

	1	2	3	4	5	6	7	8	9
1								1	
2	4								
3		2							
4					5		4		7
5			8				3		
6			1		9				
7	3			4			2		
8		5		1					
9				8		6			

Idea: $p^d_{i,j}$=true iff the value of square i, j is d

For example: $p^8_{3,5} = true$

Coding Sudoku by Propositional Clauses

- Concrete values result in units: $p^d_{i,j}$
- For every square (i, j) we generate $p^1_{i,j} \vee \ldots \vee p^9_{i,j}$
- For every square (i, j) and pair of values $d < d'$ we generate $\neg p^d_{i,j} \vee \neg p^{d'}_{i,j}$
- For every value d and column i we generate $p^d_{i,1} \vee \ldots \vee p^d_{i,9}$ (Analogously for rows and 3×3 boxes)
- For every value d, column i, and pair of rows $j < j'$ we generate $\neg p^d_{i,j} \vee \neg p^d_{i,j'}$ (Analogously for rows and 3×3 boxes)

Constraint Propagation is Unit Propagation

	1	2	3	4	5	6	7	8	9
1								1	
2	4								
3		2							
4					5		4		7
5			8				3		
6			1		9				
7	3			4	**7**		2		
8		5		1					
9				8		6			

From $\neg p^3_{1,7} \vee \neg p^3_{5,7}$ and $p^3_{1,7}$ we obtain by unit propagating $\neg p^3_{5,7}$ and further from $p^1_{5,7} \vee p^2_{5,7} \vee p^3_{5,7} \vee p^4_{5,7} \vee \ldots \vee p^9_{5,7}$ we get $p^1_{5,7} \vee p^2_{5,7} \vee p^4_{5,7} \vee \ldots \vee p^9_{5,7}$ (and finally $p^7_{5,7}$).

2.8 Other Calculi

OBDDs (Ordered Binary Decision Diagrams):

Minimized graph representation of decision trees, based on a fixed ordering on propositional variables,

⇒ canonical representation of formulas.

see script of the Computational Logic course,

see Chapter 6.1/6.2 of Michael Huth and Mark Ryan: *Logic in Computer Science: Modelling and Reasoning about Systems*, Cambridge Univ. Press, 2000.

FRAIGs (Fully Reduced And-Inverter Graphs)

Minimized graph representation of boolean circuits.

⇒ semi-canonical representation of formulas.

Implementation needs DPLL (and OBDDs) as subroutines.

Ordered resolution
Tableau calculus
Hilbert calculus
Sequent calculus
Natural deduction

see next chapter