Assignment 1 *(Propositional Logic)* (10 points)

Let $F[G \wedge H]$ be a propositional formula that contains $G \wedge H$ as a subformula (where G and H are also propositional formulas). Prove: If $F[G \wedge H]$ is valid, then $G \rightarrow F[H]$ is valid.

Assignment 2 *(Resolution)* (8 + 8 = 16 points)

Let $\Sigma = (\{a/0,\ b/0,\ f/1,\ g/1\},\ \{P/2,\ Q/1,\ R/1,\ S/1\})$; let N be the following set of clauses over Σ:

$$\neg Q(y) \ \vee\ S(x) \ \vee\ P(x,x) \ \vee\ P(y,g(y)) \qquad (1)$$
$$\neg P(z,g(a)) \ \vee\ R(z) \qquad (2)$$
$$\neg S(a) \ \vee\ \neg S(f(b)) \qquad (3)$$
$$S(f(y)) \ \vee\ S(y) \qquad (4)$$

Part (a)

Suppose that the atom ordering $\succ$ is an LPO with the precedence $P > Q > R > S > f > g > a > b$. Compute all ordered resulution inferences between the clauses (1)–(4) with respect to $\succ$. (Compute only inferences between the clauses given here, not between derived clauses. Do not compute any inferences that violate the ordering conditions of ordered resolution.)

Part (b)

If a selection function is defined appropriately, the set N is saturated under ordered resolution with selection (w.r.t. the ordering $\succ$ from Part (a)). Which literals have to be selected?

Assignment 3 *(Tableaux)* (10 points)

Use semantic tableaux to show that the following set of formulas over $\Sigma = (\{b/0,\ f/1\},\ \{P/2\})$ is unsatisfiable:

$$\forall y\, \forall x\, \Big(P(x,y) \rightarrow P(f(x),f(y))\Big)$$
$$\exists w\ P(b,w)$$
$$\forall z\, \neg\, P(f(f(b)),z)$$

Use exactly the expansion rules given in the lecture; do not use shortcuts.

Assignment 4 *(E-Algebras)* (6 + 8 + 6 = 20 points)

Let $\Sigma = (\Omega, \emptyset)$ with $\Omega = \{a/0,\ b/0,\ f/1\}$; let E be the set of equations $\{\, a \approx b,\ f(b) \approx f(f(b))\,\}$.

Part (a)

Show that $f(a) \leftrightarrow_E^* f(f(f(a)))$.

Part (b)

How many elements does the universe of the quotient algebra $\mathrm{T}_\Sigma(\emptyset)/E$ have?

Part (c)

Give an example of a (quantified) equation $\forall\vec{x}\,(t \approx t')$ such that $t \leftrightarrow_E^* t'$ does not hold, but $\mathrm{T}_\Sigma(\emptyset)/E \models \forall\vec{x}\,(t \approx t')$.

Assignment 5 *(Reduction Orderings)* (8 + 8 = 16 points)

For a signature Σ we define T_x^1 as the set of all Σ-terms that contain exactly one occurrence of the variable x and no other variables.

Part (a)

Prove: If all function symbols in Σ have arity 1, then a Knuth-Bendix ordering $\succ$ with a total precedence is total on T_x^1.

Part (b)

Prove: If Σ contains a binary function symbol and a constant function symbol, then there exists no reduction ordering that is total on T_x^1.

Assignment 6 *(Feature Vector Indexing)* (8 points)

Decide for each of the following numbers whether or not it could be used as a feature in a feature vector index:

(1) the number of ground arguments of predicate symbols in a clause,

(2) the number of variable occurrences in a clause,

(3) the number of constant symbols occurring in positive literals in a clause,

(4) the absolute value of the difference between the number of positive and the number of negative literals in a clause.

(Note on grading: A yes/no answer is sufficient; you do not have to give an explanation. However, you need at least three correct answers in this assignment to get any points. Missing answers count like false answers.)