Assignment 1 *(DPLL)* (10 points)

Let N be the following set of propositional clauses:

$$\begin{array}{ccccccccccc}
P & \vee & Q & \vee & R & \vee & S & & & & (1)\\
P & & & & & \vee & \neg S & \vee & T & & (2)\\
& & Q & & & \vee & S & & & & (3)\\
& & Q & \vee & \neg R & & & \vee & \neg T & & (4)\\
& & & & & & \neg S & \vee & T & & (5)\\
\neg P & & & \vee & R & \vee & \neg S & \vee & \neg T & & (6)\\
& & \neg Q & & & \vee & S & \vee & T & & (7)\\
& & \neg Q & & & & & \vee & \neg T & & (8)
\end{array}$$

Use the relation $\Rightarrow_{\mathrm{DPLL}}$ to test whether N is satisfiable or not; if it is satisfiable, give a model of N. Start with the "Decide" rule for the literal P, then use the "Decide" rule for the literal Q. If you use the "Backjump" rule, use the *best possible* backjump clause and go to the *best possible* successor state.

Assignment 2 *(Propositional Logic)* (10 points)

Let Π and Π' be sets of propositional variables and let μ be an injective (one-to-one) mapping from Π to Π'. For every propositional formula F over Π, let $\mu(F)$ be the formula that one obtains from F by replacing every propositional variable P in F by the propositional variable $\mu(P)$. Prove: If $\mu(F)$ is valid, then F is valid. (Note: This proof needs an induction argument; write it down in detail.)

Assignment 3 *(Propositional Logic)* (6 + 6 = 12 points)

Part (a)

Prove or refute: If F, G, and H are propositional formulas, $\neg F \vee G$ is valid, and $F \vee H$ is satisfiable, then $G \vee H$ is satisfiable.

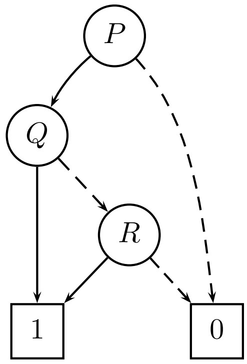
Part (b)

Prove or refute: If F, G, and H are propositional formulas, and $(F \wedge H) \rightarrow (G \wedge H)$ is valid, then $F \rightarrow G$ is valid.

Assignment 4 *(OBDDs)* (6 + 6 + 6 = 18 points)

Part (a)

Give a propositional formula F that is represented by the reduced OBDD on the right.



Part (b)

How many different reduced OBDDs over the propositional variables $\{P, Q, R\}$ have exactly one interior (non-leaf) node?

Part (c)

Find a propositional formula G over the propositional variables $\{P, Q, R\}$, such that the reduced OBDD for G has three interior nodes and the reduced OBDD for $F \lor G$ has one interior node. Give the reduced OBDDs for G and $F \lor G$.

Assignment 5 *(Algebras)* (6 + 6 + 6 = 18 points)

Let $\Sigma = (\Omega, \Pi)$ with $\Omega = \{b/0,\ f/1\}$ and $\Pi = \{p/1\}$.

Part (a)

How many different Herbrand interpretations over Σ do exist? Explain briefly.

Part (b)

How many different Herbrand models over Σ does the following formula F have?

$$p(b)\ \land\ \forall x\, \neg p(f(f(x)))$$

Part (c)

Give an example of a Σ-algebra with the universe $\{1, 2\}$ that is a model of F.

Assignment 6 *(Termination)* (12 points)

Let $\succ$ be a well-founded ordering on the set M. We define a binary relation $\rhd$ on finite subsets of M in the following way:

$$\begin{array}{lll} S \ \rhd\ S \cup \{m_1, \ldots, m_k\} & \text{if } k \geq 1,\ \{m_1, \ldots, m_k\} \subseteq M, \\ & \quad \text{and there exists an } m' \in S \\ & \quad \text{such that } m' \text{ is minimal in } S \\ & \quad \text{and } m' \succ m_i \text{ for all } i \in \{1, \ldots, k\} \\ & \\ S \ \rhd\ S \setminus \{m'\} & \text{if } m' \in S \text{ and } m' \text{ is not minimal in } S \end{array}$$

Prove that the relation $\rhd$ is terminating.