max planck institut informatik

Universität des Saarlandes
FR Informatik

Evgeny Kruglov
Christoph Weidenbach

June 15, 2010

Tutorials for "Automated Reasoning"
Exercise sheet 9

Exercise 9.1: (*3 P*)
Prove the lifting lemma for Factorisation: if C is a clause, $C\sigma$ is a ground instance of C, and

$$\frac{C\sigma}{C'}$$

is a factorisation inference, then there exists a factorisation inference

$$\frac{C}{C''}$$

and a substitution τ, such that $C' = C''\tau$.

Exercise 9.2: (*4 P*)
Let N be the following clause set:

$$\begin{array}{lclcl}
 & & Q(h(h(a))) & \vee & \neg Q(f(a)) \\
 & & Q(h(x)) & \vee & Q(f(y)) \\
\neg P(h(x), y) & \vee & Q(x) & \vee & P(h(h(x)), y) \\
\neg Q(h(a)) & \vee & P(h(a), h(h(a))) & \vee & \neg Q(a) \\
 & & & & \neg P(x, x)
\end{array}$$

Let $\succ$ be a total and well-founded ordering on ground atoms such that, if the atom A contains more symbols than B, then $A \succ B$.

a) Which literals are maximal in the clauses of N?

b) Define a selection function S such that N is already saturated under $Res_S^{\succ}$.

Clarify your hypotheses.

Exercise 9.3: (*2 P*)
Saturate the clause set $N = \{P(f(f(x))) \vee \neg P(g(x)), \neg P(f(x)) \vee P(x) \vee P(g(x)), P(a)\}$ by the general resolution calculus employing an appropriate selection function.

Hint. The plain (without selection) saturation of the given clause set is infinite. Find such a selection function, under which the general resolution calculus produces a finite saturated clause set out of the given one.

Exercise 9.4: (*2 P*)
Let N be a set of clauses, C – a clause. Prove that, if N contains a clause D, s.t. D strictly subsumes C (i.e. $C = D\sigma \vee C'$ for some substitution σ and some non-empty clause C'), then $C \in \mathit{Red}(N)$.

Exercise 9.5: (*3 P*)
Apply the congruence closure algorithm to test whether the following unit ground clauses are satisfiable:

$$f(f(a)) \approx b, f(a) \approx g(b), d \geq 6, b < c + 3, g(b) \approx c, f(c) \approx d, c \geq 2$$

Challenge Problem: (*2 Bonus Points*)
Prove Theorem 3.42(i).

Submit your solution in lecture hall 002 during the lecture on June 22. Please write your name and the date of your tutorial group (Tue, Wed, Fri) on your solution.

Note: Joint solutions are not permitted (work in groups is encouraged).