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Tutorials for "Automated Reasoning"
Exercise sheet 13

Exercise 13.1: (*4 P*)
Consider an LPO with precedence $f > g > h > a > b$ and compute R_∞ for the following ground clause set. Determine the maximal terms, literal(s) of the clauses, put the clauses in ascending order and finally compute R_∞.

$$\begin{array}{rcl} & & f(a,b) \approx h(a) \\ h(f(a,b)) \approx h(b) & \vee & f(a,b) \approx b \\ f(a,b) \not\approx a & \vee & f(h(a),b) \approx b \\ & & a \not\approx h(a) \\ g(b) \approx h(a) & \vee & g(a) \approx h(g(a)) \end{array}$$

Exercise 13.2: (*4 P*)
For the following given superposition rule and premise(s), determine the maximal literal(s) using an LPO with precedence $f > g > h > a$ and compute one conclusion if the rule is applicable. If the rule is not applicable at all, justify why. Check ordering restrictions a priori (before application of the unifier). No selection. No self inferences.

- Positive Superposition:
  $h(x) \approx g(x) \vee h(h(x)) \not\approx x \qquad f(x,y) \approx h(a) \vee h(y) \approx f(g(a), f(x,y))$
- Negative Superposition:
  $f(x,y) \approx h(y) \vee f(x,y) \not\approx y \qquad f(x,y) \approx h(x) \vee h(x) \approx g(x)$
- Equality Resolution:
  $f(y,x) \not\approx f(g(y), g(x)) \vee f(h(x), y) \not\approx h(y)$
- Equality Factoring:
  $h(f(x,z)) \approx g(x) \vee f(g(z), y) \approx h(x) \vee f(x,z) \approx z$

Exercise 13.3: (*3 P*)
Use superposition to show that the following set of (implicitly universally quantified) clauses is unsatisfiable:

$$f(a, b) \approx a$$
$$f(a, x) \approx x$$
$$f(b, c) \not\approx c \lor a \not\approx b$$

Use the KBO with weight 1 for all function symbols and variables and the precedence $f > a > b > c$; compute only inferences that are required according to the ordering restrictions of the superposition calculus.

Exercise 13.4: (*2 P*)
Consider the clause $C = C' \lor s \approx t$ where $s \approx t$ is strictly maximal in C and $s \succ t$. Prove that the conclusion of the positive superposition inference between C and (a renamed copy of) C on the top position of s is redundant.

Challenge Problem: (*2 Bonus Points*)
Prove Theorem 4.44 for the Negative Superposition rule.

Submit your solution in lecture hall 002 during the lecture on July 20. Please write your name and the date of your tutorial group (Tue, Wed, Fri) on your solution.

Note: Joint solutions are not permitted (work in groups is encouraged).