UNIVERSITÄT DES SAARLANDES
FR 6.2 – Informatik
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Lecture "Automated Reasoning" (Summer Term 2010)

Midterm Examination

Name: ....

Student Number: ....

Some notes:

- Things to do at the beginning:

  Put your student card and identity card (or passport) on the table.
  Switch off mobile phones.
  Whenever you use a new sheet of paper (including scratch paper), first write your name and student number on it.

- Things to do at the end:

  Mark every problem that you have solved in the table below.
  Stay at your seat and wait until a supervisor staples and takes your examination text.
  Note: Sheets that are accidentally taken out of the lecture room are invalid.

Sign here: **Good luck!**

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Problem	1	2	3	4a	4b	5	6a	6b	6c	7	Σ
Answered?											
Points											

Problem 1 *(Resolution)* (7 points)

Show that the following set N of clauses is unsatisfiable by deriving the empty clause via the resolution calculus.

$$N = \{A \lor \neg B,\ \neg A \lor \neg P,\ A \lor B,\ P \lor \neg A \lor \neg B,\ \neg A \lor B\}$$

Problem 2 *(DPLL)* (7 points)

Figure out whether the following set N of clauses is satisfiable by an application of the $\Rightarrow_{\text{DPLL}}$ calculus. Explicitely present backjump clauses if needed.

$$N = \{A \vee B,\ \neg P \vee Q,\ \neg Q \vee \neg A,\ \neg B \vee \neg P,\ P \vee \neg A,\ \neg B \vee P\}$$

Problem 3 *(DPLL(LA))* (8 points)

Show that the following set N of clauses is unsatisfiable by an application of the $\Rightarrow_{\text{DPLL}(LA)}$ calculus. Explicitely present backjump clauses if needed. For $\models_{LA}$ reasoning use Fourier-Motzkin.

$$N = \{\neg(z < x+5),\ x < 2 \vee y > 5,\ z < x+2 \vee \neg(y > 5),\ \neg(x < 2) \vee z < x+4\}$$

Problem 4 *(Algebras)* (5 + 3 = 8 points)

Consider the formula $F = [\forall x\,(P(x) \to P(f(x)))] \land [\forall x\, f(x) \not\approx f(f(x))]$

a) Present a model for F; show that your selected algebra models F.

b) Does F have a model over an algebra with exactly 2 elements?

Problem 5 *(CNF)* (7 points)

Transform the following formula to CNF using the optimized algorithm, i.e., in particular apply miniscoping to eventually generate small Skolem functions.

$$\forall x\, \exists y\, [R(y,a) \vee (\forall y\, (R(x,y) \leftrightarrow R(y,x)))]$$

Problem 6 *(First-Order Formulas)* (3 + 3 + 3 = 9 points)

Decide whether the following first-order formulas are satisfiable, unsatisfiable or valid, respectively. Justify your decision.

a) $(\forall x\, (P(x) \rightarrow P(f(x)))) \rightarrow P(f(a))$

b) $(\forall x\, ((P(x) \rightarrow P(f(x))) \wedge f(x) \approx g(x) \wedge P(a))) \rightarrow P(g(g(a)))$

c) $(\exists y\, P(y)) \wedge (\forall x\, (P(x) \rightarrow P(f(x)))) \wedge (\forall z\, \neg P(f(f(z))))$

Problem 7 *(First-Order Property)* (4 points)

Consider a set N of first-order ground clauses without equality. Consider a total, well-founded ordering $\prec$ on the ground atoms, that is lifted to literals by $[\neg]A \prec [\neg]B$ if ($A \prec B$ and $A \neq B$), and $A \prec \neg A$. Prove the following statement: if for all clauses in N all maximal literals are positive, then N is satisfiable.