Construction of candidate interpretations (Bachmair & Ganzinger 1990):

Let N be a set of clauses not containing $\bot$. Using induction on the clause ordering we define sets of rewrite rules E_C and R_C for all $C \in G_\Sigma(N)$ as follows:

Assume that E_D has already been defined for all $D \in G_\Sigma(N)$ with $D \prec_C C$. Then $R_C = \bigcup_{D \prec_C C} E_D$.

The set E_C contains the rewrite rule $s \to t$, if

(a) $C = C' \vee s \approx t$.

(b) $s \approx t$ is strictly maximal in C.

(c) $s \succ t$.

(d) C is false in R_C.

(e) C' is false in $R_C \cup \{s \to t\}$.

(f) s is irreducible w. r. t. R_C.

In this case, C is called *productive*. Otherwise $E_C = \emptyset$.

Finally, $R_\infty = \bigcup_{D \in G_\Sigma(N)} E_D$.

Lemma 4.45 *If $E_C = \{s \to t\}$ and $E_D = \{u \to v\}$, then $s \succ u$ if and only if $C \succ_C D$.*

Corollary 4.46 *The rewrite systems R_C and R_∞ are convergent.*

Proof. Obviously, $s \succ t$ for all rules $s \to t$ in R_C and R_∞.

Furthermore, it is easy to check that there are no critical pairs between any two rules: Assume that there are rules $u \to v$ in E_D and $s \to t$ in E_C such that u is a subterm of s. As $\succ$ is a reduction ordering that is total on ground terms, we get $u \prec s$ and therefore $D \prec_C C$ and $E_D \subseteq R_C$. But then s would be reducible by R_C, contradicting condition (f). $\square$

Lemma 4.47 *If $D \preceq_C C$ and $E_C = \{s \to t\}$, then $s \succ u$ for every term u occurring in a negative literal in D and $s \succeq v$ for every term v occurring in a positive literal in D.*

Corollary 4.48 *If $D \in G_\Sigma(N)$ is true in R_D, then D is true in R_∞ and R_C for all $C \succ_C D$.*

Proof. If a positive literal of D is true in R_D, then this is obvious.

Otherwise, some negative literal $s \not\approx t$ of D must be true in R_D, hence $s \not\downarrow_{R_D} t$. As the rules in $R_\infty \setminus R_D$ have left-hand sides that are larger than s and t, they cannot be used in a rewrite proof of $s \downarrow t$, hence $s \not\downarrow_{R_C} t$ and $s \not\downarrow_{R_\infty} t$. $\square$