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Exercise Problems for "Automated Reasoning"

Exercise 1 (*Propositional Logic*)
For a propositional formula X, let $b(X)$ be the number of occurrences of connectives ($\wedge$, $\vee$) in X. Prove using structural induction that, for every formula X, $b(X)$ = number of left parentheses in X = number of right parentheses in X. (Hence every propositional formula has the same number of left as right parentheses.)

Exercise 2 (*DPLL*)
Prove Lemma 1.9 by induction on the length of the derivation.

Exercise 3 (*DPLL*)
Let N be the following set of propositional clauses:

$$\begin{array}{ccccccccc}
\neg P & \vee & \neg Q & \vee & R & \vee & \neg S & (1) \\
 & & \neg Q & \vee & \neg R & & & (2) \\
 & & Q & \vee & \neg R & \vee & S & (3) \\
\neg P & & & \vee & R & & & (4) \\
\neg P & \vee & Q & & & \vee & \neg S & (5) \\
P & & & & & \vee & S & (6)
\end{array}$$

During a DPLL-derivation, we have reached the configuration $P^d Q^d \neg R \parallel N$.

a) Give two different backjump clauses that can be used in this situation.

b) Select one backjump clause and continue until you find a model or until you reach *fail*.

Exercise 4 (*Miniscoping*)
Apply miniscoping to the following formula:

$$\forall x \exists y \forall z (R(x,x) \wedge [P(y) \vee R(x,y) \vee Q(z)])$$

Exercise 5 (*DPLL(LA)*)
Use the transition system $\Rightarrow_{\text{DPLL}(T)}$ for the DPLL(T) procedure to check whether the following formula F is satisfiable or not. For each step explain briefly which rule you use and why. Check the T-(in)consistency of a valuation by using the Fourier-Motzkin algorithm.

$$F = \neg(((x \geq 5 \vee x < y) \wedge 3x + y \geq 20 \wedge y = x + 3) \rightarrow y \geq 9)$$

Exercise 6 (*Unification*)
Solve the following unification problems using both $\Rightarrow_{SU}$ and $\Rightarrow_{PU}$.

1. $E_1 = \{f(g(y, x_3), y, g(k(x_3), a)) \doteq f(x_1, h(x_2), x_1))\}$
2. $E_2 = \{f(x_1, h(y, g(y)), h(x_1, a)) \doteq f(g(x_2), z, z)\}$
3. $E_3 = \{f(a, x_1, g(h(y, a))) \doteq f(z, g(z), g(x_2))\}$

Exercise 7 (*Knuth-Bendix Completion*)
Apply the Knuth-Bendix completion procedure to the set of equations

$$E_0 = \{f(x, y) \approx h(x, c),\ f(x, y) \approx h(c, y),\ h(c, c) \approx c\}$$

using the lexicographic path order with the precedence $f > h > c$.

Exercise 8 (*Knuth-Bendix Completion*)
Show that the strict part $\sqsupset = \underset{\sim}{\sqsupset} \setminus \underset{\sim}{\sqsubset}$ of the encompassment quasi-ordering $\underset{\sim}{\sqsupset}$ is a well-founded strict partial ordering.

Exercise 9 (*Orderings*)
For each of the following term rewriting systems (1) to (4) give, if possible, an instance of a lexicographic path ordering and a Knuth-Bendix ordering such that the left-hand sides are larger than the right-hand sides when you compare the rules. If the rule cannot be ordered using KBO/LPO give a short argument why it can't.

$$\begin{array}{lrcl}
(1) & f(g(x), g(y)) & \rightarrow & f(f(x, g(y)), x); \\
(2) & f(g(x, a), x, a) & \rightarrow & g(f(x, x, a), h(b, x)); \\
(3) & f(x, g(y), a) & \rightarrow & f(h(x), h(y), h(a)); \\
(4) & x + 0 & \rightarrow & x \\
 & x + s(y) & \rightarrow & s(x + y).
\end{array}$$

Exercise 10 (*Superposition*)
Use the superposition calculus in order to do all possible inferences between the given clauses. Use KBO with weight 1 for all function symbols and variables and the precedence $g > f > c > b > a$.

(1) $f(f(x,y),y) \not\approx x$ and $f(a,b) \approx a \vee a \not\approx b$

(2) $f(x,y) \not\approx g(y) \vee g(x) \not\approx f(y,x)$ and $f(a,b) \approx a \vee g(a) \approx b$

(3) $f(a,y) \approx f(x,b) \vee f(x,b) \not\approx b$ and $f(x,y) \approx x$

(4) $a \approx b$ and $g(b) \approx a \vee g(b) \approx g(a)$

Exercise 11 (*Superposition*)
Consider the following clause set N:

$$\begin{array}{lr} (1) & f(a,b) \approx b \\ (2) & f(y,x) \not\approx y \\ (3) & f(f(x,y),f(y,x)) \not\approx x \vee g(x,y) \approx x \end{array}$$

1. Apply Superposition to saturate the clause set. Use KBO with weight 1 for all function symbols and variables and the precedence $g > f > c > b > a$.
2. Check yourself using SPASS: write the corresponding input file and call SPASS with the options "-PDer", "-DocProof", "-Select=0". Compare your result with the result of SPASS.

Exercise 12 (*Candidate Interpretation, Unit Rewriting*)
Consider the following clause set N. Use KBO with weight 1 for all function symbols and variables and the precedence $f > g > b > a$.

$$\begin{array}{lr} (1) & a \approx b \\ (2) & f(a,b) \approx a \\ (3) & f(b,a) \not\approx b \vee g(a) \approx b \end{array}$$

1. Which clause is false in R_∞?
2. Saturate the clauses using unit rewriting.
3. Repeat the construction of the candidate interpretation.

Exercise 13 (*Ordered Resolution with Selection*)
Refute the following clauses using the calculus of ordered resolution with selection. Take a lexicographic path order and the precedence $f > h > g > Q > R > P > a$.

$$
\begin{array}{ll}
(1) & \{R(x_1, f(a)), \neg P(g(x_1, a), y), R(h(y), f(z))\} \\
(2) & \{\neg R(x_2, x_3), Q(g(h(x_2), x_3))\} \\
(3) & \{P(g(h(x_4), x_4), x_4), Q(g(h(h(x_4)), z))\} \\
(4) & \{\neg Q(g(h(h(z)), f(z)))\}
\end{array}
$$