Universität
des
Saarlandes
FR Informatik

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Tutorials for "Automated Reasoning"
Exercise sheet 8

Exercise 8.1: (*3 P*)
Prove Theorem 3.47 from the lecture:

(i) $N \subseteq M \Rightarrow \mathit{Red}(N) \subseteq \mathit{Red}(M)$

(ii) $M \subseteq \mathit{Red}(N) \Rightarrow \mathit{Red}(N) \subseteq \mathit{Red}(N \setminus M)$

Exercise 8.2: (*2+2 P*)
Use appropriate selection functions and the general resolution calculus in order to solve the following problems.

a) Saturate the clause set: $\{\neg p(x), \neg p(g(x)), p(h(x))\}$, $\{p(b)\}$, $\{\neg p(h(x)), p(g(g(x)))\}$

b) Show that the following axioms

$$\forall x\, (x \sim x)$$
$$\forall x, y\, (x \sim y \rightarrow y \sim x)$$
$$\forall x, y, z\, (x \sim y \wedge y \sim z \rightarrow x \sim z)$$
$$\forall \vec{x}, \vec{y}\, (x_1 \sim y_1 \wedge x_2 \sim y_2 \rightarrow f(x_1, x_2) \sim f(y_1, y_2))$$

imply

$$b \sim c \wedge a \sim d \rightarrow f(a, c) \sim f(d, b)$$

Exercise 8.3: (*3 P*)
Use the Resolution Prover RP to decide whether the set of clauses given in Exercise 5.3 is satisfiable or not. For each step give the rule you use. Take the ordering $C \succ B \succ A$.

Exercise 8.4: (*2 P*)
Prove Corollary 4.5 from the lecture:
If $\rightarrow$ is normalizing and confluent, then every element b has a unique normal form.

Challenge Problem: (*2 Bonus Points*)
A relation $\rightarrow$ is *semi-confluent* iff

$$y_1 \leftarrow x \xrightarrow{*} y_2 \Rightarrow y_1 \downarrow y_2$$

Prove: A relation $\rightarrow$ is semi-confluent iff it is confluent.

Submit your solution in lecture hall 002 during the lecture on June 12. Please write your name and the date of your tutorial group (Mon, Tue, Thu) on your solution.

Note: Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.