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Tutorials for "Automated Reasoning"
Exercise sheet 3

Exercise 3.1: (*3 P*)
Let N be the following set of propositional clauses:

$$\begin{array}{ccccccccccccc}
\neg P & & & \vee & \neg R & & & \vee & \neg T & & & (1)\\
\neg P & & & & & & & \vee & T & \vee & \neg U & (2)\\
 & & & & \neg R & & & \vee & T & \vee & U & (3)\\
 & & \neg Q & \vee & \neg R & \vee & S & & & & & (4)\\
\neg P & & & \vee & R & \vee & \neg S & & & & & (5)\\
 & & Q & & & & & & & \vee & \neg U & (6)\\
P & & & & & & & & & \vee & U & (7)\\
P & \vee & \neg Q & & & & & & & \vee & \neg U & (8)
\end{array}$$

During a DPLL-derivation, we have reached the configuration $P^d Q^d R^d S \neg T U \parallel N$. Give two different backjump clauses that can be used in this situation and give the successor state with respect to $\Rightarrow_{\mathsf{DPLL}}$ for each of these backjump clauses.

Exercise 3.2: (*3 P*)
Use the Fourier-Motzkin method to decide whether the following theory is satisfiable:

$$x + y \geq 16 \qquad (1)$$
$$4x + 7y \leq 28 \qquad (2)$$
$$2x - 7y \leq 20 \qquad (3)$$
$$2x - 3y \geq -9 \qquad (4)$$

Exercise 3.3: (*3 P*)
Correctness of the Fourier-Motzkin Procedure can be formulated as follows:

$$\text{FM}(N) = \text{true iff } N \text{ is satisfiable in } \mathcal{A}_{LA}.$$

Show this property for the cases where the number n of variables in N is 0, 1 or 2.

Exercise 3.4: (*3 P*)
Use the transition system $\Rightarrow_{\text{DPLL}(T)}$ for the DPLL(T) procedure to check whether the following formula F ($x, y, z \in \mathbb{N}$) is satisfiable or not. For each step explain briefly which rule you use and why.

$$F = (z \geq 1 \rightarrow x + z \leq y) \land y + z \leq x \land ((y \leq 1 \land z \leq 1) \rightarrow x \leq y) \land z > 0$$

Challenge Problem: (*2 Bonus Points*)
The Fourier-Motzkin algorithm just returns 'yes' or 'no' to a given set N of LA atoms. Is it possible to determine the actual values which satisfy the set N? How can this be done?

Submit your solution in lecture hall 002 during the lecture on May 08. Please write your name and the date of your tutorial group (Mon, Tue, Thu) on your solution.

Note: Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.