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Tutorials for "Automated Reasoning"
Exercise sheet 1

Exercise 1.1: (*3 P*)
Determine which of the following formulas are valid/satisfiable/unsatisfiable. Use equivalence preserving transformations to transform the formulas into a CNF or DNF (don't use truth tables):

(1) $(P \wedge Q) \rightarrow (P \vee Q)$

(2) $(P \vee Q) \rightarrow (P \wedge Q)$

(3) $Q \rightarrow \neg Q$

(4) $\neg(P \rightarrow \neg P)$

(5) $\neg\big(P \vee \neg(P \wedge Q)\big)$

(6) $(P \vee \neg Q) \wedge \neg(\neg P \rightarrow \neg Q)$

Exercise 1.2: (*3 P*)
Prove or refute the following propositions:

1. Let F, G be propositional formulas and P be a propositional variable which does not occur in F nor in G. Let $F \wedge G$ be valid/satisfiable, is then $P \wedge G \wedge (P \rightarrow F)$ valid/satisfiable?

2. Let G be unsatisfiable and $F \models G$. Is then $F \vee G$ satisfiable?

Exercise 1.3: (*2 P*)
Let F, F', G, G' be propositional formulas. Prove or refute the following propositions using truth tables:

1. If $F \rightarrow (G \leftrightarrow G')$ and $G \rightarrow (F \leftrightarrow F')$ then $(F \wedge F') \leftrightarrow (G \wedge G')$ also holds.
2. If $F \rightarrow (G \leftrightarrow G')$ and $G \rightarrow (F \leftrightarrow F')$ then $(F \wedge G) \leftrightarrow (F' \wedge G')$ also holds.

Exercise 1.4: *(4 P)*
Let F', F'' be propositional formulas, P be a propositional variable. F'' is the formula which one obtains by replacing all occurrences of P by $\top$ in F'.

Show: $(P \wedge F') \models\mid (P \wedge F'')$.

Exercise 1.5: *(2 P)*
Convert the following formula to CNF following the conversion steps from the lecture.

$P \wedge \neg[(Q \leftrightarrow R) \vee (S \rightarrow T)]$

Submit your solution in lecture hall 002 during the lecture on April 24. Please write your name and the date of your tutorial group (Mon, Tue, Thu) on your solution.

Note: Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.