UNIVERSITÄT DES SAARLANDES
FR 6.2 – Informatik
Christoph Weidenbach

Lecture "Automated Reasoning"
(Summer Term 2008)

Final Examination

Name: .....

Student Number: .....

Some notes:

- Things to do at the beginning:

  Put your student card and identity card (or passport) on the table.
  Switch off mobile phones.
  Whenever you use a new sheet of paper (including scratch paper), first write your name and student number on it.

- Things to do at the end:

  Mark every problem that you have solved in the table below.
  Stay at your seat and wait until a supervisor staples and takes your examination text.
  Note: Sheets that are accidentally taken out of the lecture room are invalid.

Sign here: **Good luck!**

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Problem	1	2	3a	3b	4	5	6	7	Σ
Answered?									
Points									

Problem 1 *(DPLL(LA))* (10 points)

Refute the unit clauses $x < 7$, $x' \leq x + 1$, , $x' \geq x + 1$, $x \leq 5$, $x' \geq 7$ via DPLL(LA) using the Fourier-Motzkin procedure.

Problem 2 *(Ordered Resolution)* (10 points)

Refute the following clause set via ordered resolution using a KBO where all signature symbols and variables have weight 1 and the precedence is $P > Q > R > g > a$. As literal ordering use the multiset $\{A\}$ for each atom A and the multiset $\{B, B\}$ for each negative literal $\neg B$. Selection is not permitted.

$$P(a, z) \tag{1}$$
$$\neg Q(z) \ \vee \ \neg P(z, a) \tag{2}$$
$$R(a) \tag{3}$$
$$\neg R(v) \ \vee \ Q(g(v)) \tag{4}$$
$$\neg P(x, g(y)) \ \vee \ P(g(x), y) \tag{5}$$

Problem 3 *(Orderings)* (6 + 6 = 12 points)

Part (a) For the following term pairs, find if possible a precedence for the LPO such that the left term gets larger than the right term. If the terms cannot be ordered using the LPO, please provide a justification.

- $f(g(x), h(y))$, $g(f(h(x), h(y)))$
- $f(f(x, y), g(z))$, $f(g(z), f(x, y))$
- $g(f(x, y))$, $g(g(h(x)))$

Part (b) For the following term pairs, find if possible a precedence and weighting function for the KBO such that the left term gets larger than the right term. If the terms cannot be ordered using the KBO, please provide a justification.

- $f(f(x, y), g(z))$, $f(h(z), f(x, h(y)))$
- $f(g(x), h(y))$, $g(f(h(y), h(x)))$
- $g(f(x, y))$, $h(g(f(x, x)))$

Problem 4 *(Superposition)* (16 points)

For the following given superposition rule and premise(s), determine the maximal literal(s) using an LPO with precedence $f > g > h > a$ and compute one conclusion if the rule is applicable. If the rule is not applicable at all, justify why. Check ordering restrictions a priori (before application of the unifier). No selection. No self inferences.

- Positive Superposition:
  $g(x) \approx h(x) \ \vee \ f(x,y) \approx h(a) \qquad f(x,y) \not\approx y \ \vee \ f(g(x),y) \approx h(y)$
- Negative Superposition:
  $f(x,y) \not\approx y \ \vee \ g(f(x,y)) \approx h(y) \qquad h(x) \not\approx g(x) \ \vee \ f(y,z) \not\approx h(a)$
- Equality Resolution:
  $g(x) \not\approx h(x) \ \vee \ f(y,z) \not\approx f(a,h(x))$
- Equality Factoring:
  $g(f(x,z)) \approx g(y) \ \vee \ f(g(z),y) \approx h(x) \ \vee \ f(x,y) \approx y$

Problem 5 *(Redundancy)* (10 points)

Prove that replacing a clause $C \ \vee \ x \not\approx t$ where x does neither occur in C nor in t by the subclause C is an instance of the superposition redundancy criterion.

Problem 6 *(Model Construction)* (12 points)

Consider an LPO with precedence $f > g > h > a > b$ and compute R_∞ for the following ground clause set. Determine the maximal terms, literal(s) of the clauses, put the clauses in ascending order and finally compute R_∞.

$$f(g(a),b) \approx h(a)$$
$$f(a,b) \approx b \ \vee \ f(a,b) \approx b$$
$$f(g(a),b) \not\approx b \ \vee \ f(h(a),b) \approx b$$
$$a \not\approx b$$
$$g(a) \approx h(b) \vee g(b) \approx h(a)$$

Problem 7 *(Model Properties)* (10 points)

Prove that for any satisfiable, saturated clause set $N = N' \cup \{x \approx a \ \vee \ x \approx b\}$ the set $T_\Sigma(\emptyset)/R_\infty$ has at most two elements.