UNIVERSITÄT DES SAARLANDES
FR 6.2 – Informatik
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Lecture "Automated Reasoning"
(Summer Term 2008)

Midterm Examination

Name: ....

Student Number: ....

Some notes:

- Things to do at the beginning:

  Put your student card and identity card (or passport) on the table.
  Switch off mobile phones.
  Whenever you use a new sheet of paper (including scratch paper), first write your name and student number on it.

- Things to do at the end:

  Mark every problem that you have solved in the table below.
  Stay at your seat and wait until a supervisor staples and takes your examination text.
  Note: Sheets that are accidentally taken out of the lecture room are invalid.

Sign here: **Good luck!**

....

Problem	1	2a	2b	3	4	5a	5b	6	7	8	Σ
Answered?											
Points											

Problem 1 *(DPLL)* (10 points)

Consider the propositional clause set

$$N' = N \cup \{\neg A_1 \vee \neg A_4 \vee A_6,\ \neg A_1 \vee \neg A_4 \vee \neg A_6\}$$

During a DPLL-derivation, we have reached the state $A_1^d\, A_2^d\, \neg A_3\, A_4^d\, A_6 \parallel N'$. Give two different backjump clauses that can be used in this situation and give the successor state with respect to $\Rightarrow_{\text{DPLL}}$ for each of these backjump clauses.

Problem 2 *(Algebras)* (6 + 6 = 12 points)

Let $\Sigma = (\Omega, \Pi)$, where $\Omega = \{a, b, c\}$ and $\Pi = \{P\}$. Let N be the set of formulas $\{\forall x \exists y P(x, y), \neg P(a, b), \neg P(a, c)\}$.

Part (a) Give a Σ-algebra that is a model of N.

Part (b) Does N have a model over the universe $\{1, 2\}$? If yes, present the appropriate Σ-algebra. If no, prove why such a model cannot exist.

Problem 3 *(CNF)* (10 points)

Transform the forumla

$$\forall x \exists y \forall z (R(x, x) \vee (P(y) \wedge R(x, y) \wedge Q(z)))$$

into CNF using miniscoping.

Problem 4 *(Unification)* (10 points)

Transform the following unification problem into solved form using either $\Rightarrow_{SU}$ or $\Rightarrow_{PU}$:

$$E = \left\{\ f\big(x, g(h(y, z)), g(g(b))\big) \doteq f\big(g(h(a, g(y))), x, g(z)\big)\ \right\}.$$

Problem 5 *(Model Construction)* (6 + 6 = 12 points)

Consider the following ground clause set N

$$
\begin{aligned}
&P(a,a)\\
&\neg Q(a) \;\lor\; \neg P(a,a)\\
&R(a)\\
&\neg R(a) \;\lor\; Q(g(a))\\
&\neg P(a,g(a)) \;\lor\; P(g(a),a)
\end{aligned}
$$

with atom ordering $R(a) \succ P(g(a),a) \succ P(a,g(a)) \succ P(a,a) \succ Q(g(a)) \succ Q(a)$.

Part (a) Construct I_N.

Part (b) Determine the minimal clause not satisfied by I_N and perform one ordered ground resolution step with that clause generating a smaller clause not satisfied by I_N.

Problem 6 *(Resolution)* (10 points)

Refute the following clause set via general resolution.

$$
\begin{aligned}
&P(a,b) && (1)\\
&\neg P(x,y) \;\lor\; P(y,x) && (2)\\
&\neg P(x,y) \;\lor\; P(f(x),y) && (3)\\
&\neg P(b,f(f(a))) && (4)
\end{aligned}
$$

For each inference give the parent clause numbers and the resulting clause.

Problem 7 *(Clause Sets)* (10 points)

A clause is called *positive* if it consists of positive literals only, i.e., atoms. Let N be a first-order clause set that does not contain a positive clause. Prove that N is satisfiable.

Problem 8 *(Terms)* (10 points)

Let $\# : T_\Sigma \mapsto \mathbb{N}$ be a function mapping ground terms to the number of symbols occurring in the term, e.g., $\#(g(a)) = 2$, $\#(h(a, g(b))) = 4$. Furthermore, let $>>$ be a total ordering on Ω. Now consider the binary relation $\succ \subset T_\Sigma \times T_\Sigma$ defined by $t \succ s$ where $t = f(t_1, \ldots, t_n)$, $s = g(s_1, \ldots, s_m)$ iff

1. $\#(t) > \#(s)$ or
2. $\#(t) = \#(s)$ and $f >> g$ or
3. $\#(t) = \#(s)$, $f = g$ and $(t_1, \ldots, t_n) \succ_{\text{lex}} (s_1, \ldots, s_m)$

Prove by structural induction on the ground terms that $\succ$ is total.