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Tutorials for "Automated Reasoning"
Exercise sheet 3

Exercise 3.1: (*3 P*)
Consider the clauses $\{\neg P \lor \neg Q \lor R, \neg S \lor T, \neg P \lor \neg T, \neg Q \lor \neg R \lor V, \neg P \lor \neg R \lor \neg Q \lor \neg V, Q \lor T\} \subset N$ and assume you derived already via $\Rightarrow_{\mathsf{DPLL}}$ the configuration $PQ^dRS^dT \parallel N$. Prove that starting from this configuration, N is unsatisfiable via $\Rightarrow_{\mathsf{DPLL}}$.

Exercise 3.2: (*3 P*)
Formulate SplitHornSAT as a rule based system (define $\Rightarrow_{\mathsf{SplitHornSAT}}$). Tip: Probably, the configuration used for DPLL (*Literals* $\parallel$ *Clauses*) is not appropriate.

Exercise 3.3: (*2 P*)
There is another subclass of propositional CNFs where satisfiability can be decided in polynomial time: the class 2SAT where every clause has at most 2 literals.

a) Scetch the algorithm that decides 2SAT in polynomial time.

b) Why does this algorithm not apply to 3SAT (every clause has at most 3 literals)?

Exercise 3.4: (*2 P*)
We call a clause C *superfluous* in $N \cup \{C\}$ if $N \models C$. Now consider the clause encoding of the SUDOKU puzzle.

a) Prove that the encoding contains superfluous clauses.

b) Would it make sense to remove the superfluous clauses?

Challenge Problem: (*2 Bonus Points*)
Assume a Horn SAT problem N and a propositional variable A. We want to test whether $N \models A$. For that purpose we employ the following rules on a set of negative literals M:

a) $M \cup \{L\} \Rightarrow_{\mathsf{BWD}} M \qquad$ if $\overline{L} \in N$

b) $M \cup \{L\} \Rightarrow_{\mathsf{BWD}} M \cup \{\neg P_1, \dots, \neg P_n\} \qquad$ if $P_1, \dots, P_n \rightarrow \overline{L} \in N$

Now we check $N \models A$ by searching for a derivation $\{\neg A\} \Rightarrow_{\mathsf{BWD}} \dots \Rightarrow_{\mathsf{BWD}} \emptyset$. What are the problems with this approach?

Submit your solution in lecture hall 003 during the lecture on May 15. Please write your name and the date of your tutorial group (Mon, Thu, Fri) on your solution.

Note: Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.