



Uwe Waldmann
Christoph Weidenbach

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Tutorials for “Automated Reasoning”
Exercise sheet 2

Exercise 2.1: (2 P)

Prove: A formula in CNF is valid, if and only if each of its disjunctions contains a pair of complementary literals P and $\neg P$.

Exercise 2.2: (2 P)

Prove: Let P be a propositional variable. If the formula $F[P] \wedge (P \leftrightarrow F')$ is satisfiable, then $F[F']$ is satisfiable. (The reverse implication was shown in the lecture.)

Exercise 2.3: (1 + 2 P)

Let F and G be propositional formulas. Prove or refute:

- (1) If F is satisfiable or G is satisfiable, then $F \vee G$ is satisfiable.
- (2) If F is satisfiable and G is satisfiable, then $F \wedge G$ is satisfiable.

Exercise 2.4: (1 P)

Suppose you have a propositional formula F (not necessarily in CNF or DNF) and you want to check its validity using an implementation of the DPLL procedure. What do you have to do and in what order?

Exercise 2.5: (2 P)

Use the transition system $\Rightarrow_{\text{DPLL}}$ for the DPLL procedure to check whether the following set of propositional clauses is satisfiable or not. For each step explain briefly which rule you use and why.

$$\neg P \quad \quad \quad \vee \quad \neg R \quad \quad \quad (1)$$

$$\neg P \quad \quad \quad \quad \quad \quad \vee \quad S \quad (2)$$

$$P \vee \neg Q \vee \neg R \quad (3)$$

$$P \vee \neg Q \vee R \quad (4)$$

$$P \vee Q \vee \neg R \quad (5)$$

$$Q \vee R \vee S \quad (6)$$

$$R \vee \neg S \quad (7)$$

Challenge Problem: (2 Bonus Points)

Let N be an unsatisfiable set of clauses over $\{P, Q, R\}$, such that there is no unsatisfiability proof using $\Rightarrow_{\text{DPLL}}$ that needs less than 2 applications of “Decide”. How many clauses does N contain at least? Give an explanation.

Submit your solution in lecture hall 003 during the lecture on May 8. Please write your name and the date of your tutorial group (Mon, Thu, Fri) on your solution.

Note: Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.