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Tutorials for "Automated Reasoning"
Exercise sheet 10

Exercise 10.1: *(2 P)*
Use the polynomial interpretation $\mathcal{A}$ with $U_{\mathcal{A}} = \mathbb{N} \setminus \{0, 1, 2\}$ and $P_f := X^2 + XY$ to show that the term rewriting system

$$\begin{array}{rcl} f(f(x,y),z) & \to & f(x, f(y,z)) \\ f(y, f(x,z)) & \to & f(x,x) \end{array}$$

terminates.

Exercise 10.2: *(6 P)*
Find polynomial orderings that prove the termination of the following term rewriting systems:

(1) $$f(g(x)) \to g(g(f(x)));$$

(2) $$h(f(x), g(x)) \to h(g(x), f(x));$$

(3) $$(x + y) \cdot z \to (x \cdot z) + (y \cdot z);$$

(4) $$\begin{array}{rcl} x + 0 & \to & x \\ x + s(y) & \to & s(x + y) \\ d(0) & \to & 0 \\ d(s(x)) & \to & s(s(d(x))); \end{array}$$

(5) $$\begin{array}{rcl} (x + y) + z & \to & x + (y + z) \\ f(x) + f(y) & \to & f(x + y); \end{array}$$

(6) $$\begin{array}{rcl} (\neg x) \supset y & \to & x \vee y \\ x \supset (y \vee z) & \to & y \vee (x \supset z) \\ (\neg x) \supset (y \supset z) & \to & y \supset (x \vee z). \end{array}$$

Exercise 10.3: (*2 P*)
Prove that the following term rewrite system is confluent:

$$\begin{array}{rcl} f(g(x)) & \to & x \\ g(f(x)) & \to & x \\ f(b) & \to & c \\ b & \to & g(c) \end{array}$$

Challenge Problem: (*3 Bonus Points*)
Give an example of

- a signature Σ, containing at least one constant symbol,
- a set E of equations over Σ,
- two terms s, t over Σ,

with the property that $T_\Sigma(\{x_1\})/E \models \forall\vec{x}(s \approx t)$, but $T_\Sigma(\{x_1, x_2\})/E \not\models \forall\vec{x}(s \approx t)$ (where $\vec{x}$ consists of all the variables occurring in s and t).

Submit your solution in lecture hall 003 during the lecture on July 3. Please write your name and the date of your tutorial group (Mon, Thu, Fri) on your solution.

Note: Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.