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Tutorials for "Automated Reasoning"
Exercise sheet 1

Exercise 1.1: (*3 P*)
Determine which of the following formulas are valid/satisfiable/unsatisfiable:

(1) $(P \wedge Q) \rightarrow (P \vee Q)$

(2) $(P \vee Q) \rightarrow (P \wedge Q)$

(3) $\neg(P \wedge \neg\neg P)$

(4) $Q \rightarrow \neg Q$

(5) $Q \wedge \neg Q$

(6) $\neg(\neg P \vee \neg\neg P)$

(7) $\big((P \rightarrow Q) \wedge (\neg P \rightarrow R)\big) \rightarrow (Q \vee R)$

Exercise 1.2: (*2 P*)
Prove Prop. 1.4: If N is a set of propositional formulas, then $N \models F$ if and only if $N \cup \{\neg F\}$ is unsatisfiable. (A set of propositional formulas is unsatisfiable, if and only if for every valuation $\mathcal{A}$ there is a formula G in the set such that $\mathcal{A} \not\models G$.)

Exercise 1.3: (*5 P*)
Let F be a propositional formula which contains no occurrence of $\rightarrow$ or $\leftrightarrow$, then F° is the propositional formula obtained by replacing all occurrences of propositional variables by their negations. For instance, if $F = (P \vee \neg Q)$, then $F^\circ = (\neg P \vee \neg\neg Q)$.

Let F be a propositional formula which contains no occurrence of $\rightarrow$ or $\leftrightarrow$. The *dual* of F, which we denote here by F^*, is the propositional formula obtained by replacing every occurrence of $\top$ by $\bot$, every occurrence of $\bot$ by $\top$, every occurrence of $\vee$ by $\wedge$ and every occurrence of $\wedge$ by $\vee$.

(1) Prove that a propositional formula F which contains no occurrence of $\rightarrow$ or $\leftrightarrow$ is valid if and only if F° is valid.

(2) Prove: $F^* \models\!\!\!\dashv \neg F^\circ$.

(3) Prove that for all formulas F, G which contain no occurrences of $\rightarrow$ or $\leftrightarrow$, $\models F \leftrightarrow G$ if and only if $\models F^* \leftrightarrow G^*$. *(The duality principle)*

Submit your solution in lecture hall 003 during the lecture on May 3. Please write your name and the date of your tutorial group (Mon, Thu, Fri) on your solution.

Note: Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.