INFORMATIK

Universität des Saarlandes
FR Informatik

Harald Ganzinger
Uwe Waldmann

May 16, 2002

Tutorials for "Logic in Computer Science"
Exercise sheet 6

Exercise 6.1:
Let N be a set of propositional formulas, let C be a propositional formula, such that $N \models C$. Then there exists a finite subset $M \subseteq N$ such that $M \models C$.

Exercise 6.2:
Use resolution to show that the following set of clauses is unsatisfiable:

$$p(a, z)$$
$$\neg p(f(f(a)), a)$$
$$\neg p(x, g(y)) \vee p(f(x), y)$$

Exercise 6.3:
When a resolution theorem prover finds a refutation of a set of clauses, then it has usually generated much more clauses than those which are actually required for the proof: On the one hand, many partial proof attempts can not be extended to a complete proof; on the other hand, it may happen that the same formula has been derived over and over again in different ways. Unfortunately, there is no exact method to tell in advance whether a newly generated clause is necessary or superfluous. However, the concept of redundancy offers the possibility to delete at least *some* of the superfluous clauses without losing refutational completeness:

> **Definition:** A ground clause C is called redundant with respect to a set N of clauses, if there exist clauses $D_1, \ldots, D_k$ in $G_\Sigma(N)$ such that $D_i \prec C$ for every $i \in \{1, \ldots, k\}$ and $\{D_1, \ldots, D_k\} \models C$.
>
> A (possibly non-ground) clause C is called redundant with respect to a set N of clauses, if every ground instance $C\sigma$ of C is redundant with respect to N.
>
> The set of all clauses that are redundant with respect to a set N of clauses is denoted by $\mathrm{Red}(N)$. (Note: a clause that is redundant with respect to N need not be contained in N.)

Prove the following lemma: If N is a saturated set of ground clauses not containing $\bot$, and if $C \in N$ is redundant with respect to N, then $I_C \models C$. (Consequently, C is neither productive nor a minimal counterexample.)

Exercise 6.4:
Redundant clauses remain redundant, if the theorem prover derives new clauses and adds them to the current set of clauses, or if it deletes redundant clauses. Prove:

(a) If N and M are sets of clauses and $N \subseteq M$, then $\mathrm{Red}(N) \subseteq \mathrm{Red}(M)$.

(b) If N and M are sets of clauses and $M \subseteq \mathrm{Red}(N)$, then $\mathrm{Red}(N) \subseteq \mathrm{Red}(N \setminus M)$.

(Hint: Part (b) is a bit tricky. Use Exercise 6.1 and the fact that the multiset extension of a well-founded ordering is again well-founded.)

Exercise 6.5:
Let N be a set of (possibly non-ground) clauses, let C be a (possibly non-ground) clause. Show:

(a) If C is a tautology, then $C \in \mathrm{Red}(N)$.

(b) If N contains a clause D such that D *strictly subsumes* C (that is, $C = D\sigma \vee C'$ for some substitution σ and some non-empty clause C'), then $C \in \mathrm{Red}(N)$.

Put your solution into the mail box at the door of room 627 in the MPI building (46.1) before May 24, 11:00 (Group D: before May 27, 11:00). Don't forget to write your name and the name of your tutorial group (B, C, D) on your solution.