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**Tutorials for “Logic in Computer Science”**  
**Exercise sheet 5**

**Exercise 5.1:**

Prove part (iv) of Proposition 1.19: Let  $D'$ ,  $D$ , and  $C$  be ground clauses such that  $D, D' \in N$  and  $D' \succ D \succ C$ . Then

$$I_D \models C \Rightarrow I_{D'} \models C \text{ and } I_N \models C.$$

If, in addition,  $C \in N$  or  $\max(D) \succ \max(C)$ :

$$I_D \not\models C \Rightarrow I_{D'} \not\models C \text{ and } I_N \not\models C.$$

**Exercise 5.2:**

Explain the importance of Proposition 1.19, part (v) for the proof of the model existence theorem.

**Exercise 5.3:**

Prove the following statement: If  $(M, \succ)$  is a well-founded total ordering and  $M$  is infinite, then there exists a subset  $M' \subseteq M$  such that  $(M', \succ)$  and  $(\mathbb{N}, >)$  are order-isomorphic.

**Exercise 5.4:**

Compute a most general unifier of

$$\{ f(x, g(x)) \doteq y, h(y) \doteq h(v), v \doteq f(g(z), w) \}$$

**Exercise 5.5:**

Implement a function `cnf` in ML that takes a quantifier-free formula (that is, the matrix of a prenex formula), and computes its clausal normal form.

Put your solution into the mail box at the door of room 627 in the MPI building (46.1) before May 16, 11:00 (Group D: before May 21, 11:00). Don't forget to write your name and the name of your tutorial group (B, C, D) on your solution.