INFORMATIK

Universität des Saarlandes
FR Informatik

Harald Ganzinger
Uwe Waldmann

May 2, 2002

Tutorials for "Logic in Computer Science"
Exercise sheet 4

Exercise 4.1:
Skolemization preserves the (un-)satisfiability of prenex formulas. Show that it does not preserve validity: Give a signature $\Sigma = (\Omega, \Pi)$ and a closed prenex Σ-formula F without equality, such that F is valid, but the formula F' that is obtained by Skolemizing F is not valid.

Exercise 4.2:
Theorem 1.10 holds for (universally quantified) clauses. Show that it does not hold for arbitrary Σ-formulas: Give a signature $\Sigma = (\Omega, \Pi)$, such that Ω contains at least one constant symbol, and a closed Σ-formula F without equality, such that F is satisfiable but does not have a Herbrand model over Σ. (Why does transformation into clause normal form change this situation?)

Exercise 4.3:
Ackermann's function $ack(m, n)$ is defined using the following equations:

$$\begin{aligned} ack(0, n) &= n + 1 \quad \text{for } n \in \mathbb{N} \\ ack(m + 1, 0) &= ack(m, 1) \quad \text{for } m \in \mathbb{N} \\ ack(m + 1, n + 1) &= ack(m, ack(m + 1, n)) \quad \text{for } m, n \in \mathbb{N} \end{aligned}$$

Use Noetherian induction to show that the recursive computation of $ack(m, n)$ terminates for all $m, n \in \mathbb{N}$.

Exercise 4.4:
Two ordered sets $(M_1, \succ_1)$ and $(M_2, \succ_2)$ are called order-isomorphic, if there exists a bijection $h : M_1 \to M_2$ such that for all $x, y \in M_1$: $x \succ_1 y$ if and only if $h(x) \succ_2 h(y)$.

Let the ordering $\succ$ over $\{A, B\}$ be defined by $A \succ B$. Is the set of finite multisets over $\{A, B\}$ (ordered by the multiset extension of $\succ$) order-isomorphic to $(\mathbb{N}, >)$ or not? Prove your hypothesis.

Exercise 4.5:
Find a total ordering $\succ$ on the ground atoms A, B, C, D, E, such that the associated clause ordering $\succ_C$ orders the following clauses like this:

$$B \vee C \ \succ_C \ A \vee A \vee \neg C \ \succ_C \ C \vee E \ \succ_C \ C \vee D \ \succ_C \ \neg A \vee D \ \succ_C \ \neg E$$

Exercise 4.6:
Prove that the multiset extension $\succ_{mul}$ of a total ordering $\succ$ is total.

Put your solution into the mail box at the door of room 627 in the MPI building (46.1) before May 10, 11:00 (Group D: before May 13, 11:00). Don't forget to write your name and the name of your tutorial group (B, C, D) on your solution.