INFORMATIK

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des
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FR Informatik

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Tutorials for „Logic in Computer Science“
Exercise sheet 2

Exercise 2.1:
Prove or refute the following statements:

(a) If F is a first-order formula, then F is valid if and only if $F \rightarrow \bot$ is unsatisfiable.

(b) If F is a first-order formula and x a variable, then F is unsatisfiable if and only if $\exists x F$ is unsatisfiable.

(c) If F and G are first-order formulas, F is valid, and $F \rightarrow G$ is valid, then G is valid.

(d) If F and G are first-order formulas, F is satisfiable, and $F \rightarrow G$ is satisfiable, then G is satisfiable.

(e) If F and G are first-order formulas and $F \models G$, then $F \models \neg G$ does not hold.

(f) If F and G are first-order formulas and $F \models G$, then $\neg F \models G$ does not hold.

(g) If F, G, and H are first-order formulas and $F \wedge G \models H$, then $F \models H$.

(h) If F, G, and H are first-order formulas and $F \vee G \models H$, then $F \models H$.

Exercise 2.2:
Find a first-order formula F such that

(a) F is satisfiable and all models of F have exactly two elements.

(b) F is satisfiable and all models of F are infinite. (Hint: consider the definition of an ordering.)

Exercise 2.3:
Show that the prenex form of a first-order formula that is computed by the rewrite relation $\Rightarrow_P$ may contain more quantifiers than the formula itself.

Exercise 2.4:
Implement the following functions in ML:

(a) `closed : wff -> bool` tests whether a formula is closed (i. e., whether all occurrences of variables are bound).

(b) `substitute : wff -> term -> vname -> wff` takes a formula F, a term t, and a variable x as arguments and computes $F[t/x]$.

Put your solution into the mail box at the door of room 627 in the MPI building (46.1) before April 25, 11:00. Don't forget to write your name and the name of your tutorial group (B, C, D) on your solution.