INFORMATIK

Universität
des
Saarlandes
FR Informatik

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Tutorials for “Logic in Computer Science”
Exercise sheet 11

Exercise 11.1:
We represent relationships beween students, lectures, tutorial groups, and tutors using a list of terms in the following way:

```
[ visits(eva, logic, 3),      % Eva visits tutorial group 3 of the logic lecture
  visits(jan, logic, 1),      % Jan visits tutorial group 1 of the logic lecture
  visits(jan, ai, 2),         % Jan visits tutorial group 2 of the AI lecture
  tutors(eva, net, 1),        % Eva tutors group 1 of the networks lecture
  visits(ali, net, 1),        % Ali visits tutorial group 1 of the networks lecture
  tutors(ali, logic, 3) ]     % Ali tutors group 3 of the logics lecture
```

Implement a Prolog predicate `tutormutually(`l`,`x`,`y`)` that takes such a list l and finds students x and y that visit each other's tutorial groups (e. g., Eva and Ali in the example above).

Exercise 11.2:
Give an example of a set U and a function $f : 2^U - > 2^U$ such that f is monotone but not continuous.

Hint: U must be infinite, e. g., the natural numbers. (Why?)

Challenge: Find a monotone function f such that $\bigcup_{i=0}^{\infty} f^i(\emptyset)$ is not a fixpoint of f.

Exercise 11.3:
Prove or refute the following LTL statements:

(a) $\models \mathsf{G}(p \rightarrow q) \rightarrow (\mathsf{F}p \rightarrow \mathsf{F}q)$

(b) $\models \mathsf{F}(p \rightarrow q) \rightarrow (\mathsf{F}p \rightarrow \mathsf{F}q)$

(c) $\models \mathsf{G}(p \leftrightarrow \mathsf{X}p) \leftrightarrow (\mathsf{G}p \vee \mathsf{G}\neg p)$

(Statements (a) and (b) are easy, (c) is a bit more involved.)

Exercise 11.4:
The behaviour of an elevator can be described using the following set of nullary predicates (propositional variables):

up	elevator is moving upwards
down	elevator is moving downwards
halting	elevator is not moving
at_i	elevator is at floor i ($0 \leq i \leq 2$)
$above_i$	elevator is between floors i and $i+1$ ($0 \leq i \leq 1$)
$activated_i$	a button for floor i is activated ($0 \leq i \leq 2$) (i. e., somebody wants to enter or leave at floor i)

Express the following statements in LTL:

(a) The elevator cannot move upwards forever.

(b) If the elevator is moving upwards at some time and moving downwards at some later time, then it must have stopped in between.

(c) If the elevator is between floors 0 and 1 in some state, then in the next state it is either still between floors 0 and 1, or at floor 0, or at floor 1.

(d) Whenever the button for floor 2 is activated, the elevator will stop there eventually. The button remains activated, until the elevator stops there, and then it becomes non-activated. While the button is activated, the elevator does not pass by floor 2 without halting.

Put your solution into the mail box at the door of room 627 in the MPI building (46.1) before July 5, 11:00 (Group D: before July 8, 11:00). Don't forget to write your name and the name of your tutorial group (B, C, D) on your solution.