Universität
des
Saarlandes
FR Informatik

MPI INFORMATIK

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Tutorials for "Logic in Computer Science"
Exercise sheet 10

Exercise 10.1:
We define a matching relation between lists (analogously to the matching relation for Unix shell patterns) in the following way:

- every list matches itself.
- the list [$*$] matches every (possibly empty) list.
- the list [?] matches every list of length 1.
- if l_1 matches l_1' and l_2 matches l_2', then the concatenation of l_1 and l_2 matches the concatenation of l_1' and l_2'.

Example: The list $[*, \mathrm{one}, *, *, 5, ?]$ matches the list $[\mathrm{one}, 2, 3, 4, 5, \mathrm{six}]$.

Implement a Prolog predicate `matches(XL,YL)` that succeeds if and only if the list `XL` matches the list `YL`. (Hint: One can implement `matches` without using `append` or other auxiliary predicates. If you prefer to use `append`, beware of non-termination.)

Exercise 10.2:
Give a logical program P such that $T_P^0(\emptyset) \subset T_P^1(\emptyset) \subset T_P^2(\emptyset) \subset T_P^3(\emptyset) = T_P^n(\emptyset)$ for all $n \geq 3$.

Exercise 10.3:
Let $\Sigma = (\{a/0\}, \{p/1, q/1\})$. Show that there exists no Σ-model I of the clause $C = p(a) \vee q(a)$ such that $I \models A \;\Leftrightarrow\; C \models A$ for all Σ-atoms A. (In other words: C does not have a canonical model.)

Exercise 10.4:
Prove part (1) of Theorem 2.9:

$$I_P \;=\; I_{G_\Sigma(P)} \;=\; \bigcup\nolimits_{i=0}^{\infty} T_P^i(\emptyset) \;=\; \bigcap\nolimits_{T_P(I)\subseteq I} I$$

You can use all the preceding theorems.

Exercise 10.5:
Prove the footnote on slide 27: Let Σ be a signature with infinitely many constants. Let P be a set of closed Σ-formulas, let $\forall \vec{x}\, G$ be a closed Σ-formula (where $\vec{x}$ is the list of all variables occurring in G). Let ρ be a substitution that maps all variables in $\vec{x}$ to pairwise different constants that do not occur in P or G. Then $P \models G\rho$ implies $P \models \forall \vec{x}\, G$.

Put your solution into the mail box at the door of room 627 in the MPI building (46.1) before June 28, 11:00 (Group D: before July 1, 11:00). Don't forget to write your name and the name of your tutorial group (B, C, D) on your solution.