INFORMATIK

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FR Informatik

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Tutorials for „Logic in Computer Science"
Exercise sheet 1

Exercise 1.1:
Let $\Sigma = (\Omega, \Pi)$ be a signature, where $\Omega = \{f/2,\ g/1,\ a/0,\ b/0\}$ and $\Pi = \{p/1\}$; let X be the set of variables $\{x, y, z\}$. Which of the following expressions are terms over Σ and X, which are atoms/literals/clauses/formulae, which are neither?

(a) $\neg p(f(x,y))$

(b) $f(x,x) \approx x$

(c) $g(f(a,b))$

(d) $p(g(x)) \vee p(x)$

(e) $p(\neg g(x))$

(f) $p(a) \wedge p(b) \wedge y$

(g) $\exists y(\neg p(f(y,y)))$

(h) $\forall x(g(p(x)) \approx g(x))$

Exercise 1.2:
Formalize the following statements in the signature Σ_{PA} of Peano arithmetic:

(a) 3 is not divisible by 2.

(b) All numbers between 1 and 3 are even.

(c) There exists exactly one number between 1 and 3.

(d) There does not exists a largest square number.

Exercise 1.3:
Compute the results of the following substitutions:

(a) $f(g(x), x)[g(a)/x]$

(b) $p(f(y, x))[x/y]$

(c) $\forall y(p(f(y, x)))[x/y]$

(d) $\forall y(p(f(y, x)))[y/x]$

(e) $\forall y(p(f(z, g(y))) \vee \exists z(g(z) \approx y))[g(b)/z]$

(f) $\exists y(f(x, y) \approx x \rightarrow \forall x(f(x, y) \approx x))[g(y)/y, g(z)/x]$

Exercise 1.4:
Prove or refute the following statement:

If t, s, s' are terms and x and y are distinct variables, then $(t[s/x])[s'/y] = t[s/x, s'/y]$.

Exercise 1.5:
At `http://www.mpi-sb.mpg.de/~uwe/lehre/logic/programs/` you can find the following two files:

`pl1_syntax.ml`: contains the definition of an SML datatype `term` for terms over arbitrary signatures.

`exercise1.ml`: contains the definition of a function `termvars` that computes the list of all variables occurring in an argument of the type `term`.

Extend `pl1_syntax.ml` with the definition of SML datatypes `atom` for atoms and `wff` for formulae; extend `exercise1.ml` with the definition of functions `atomvars` and `formulavars` that compute the list of all variables occurring in an atom or in a formula, respectively.

Put your solution into the mail box at the door of room 627 in the MPI building (46.1) before April 18, 11:00. Don't forget to write your name and the name of your tutorial group (B, C, D) on your solution.