Problem 1 *(Algebras and semantics)* (4 + 4 = 8 points)

Part (a)

Prove the following statement: If F and G are first-order formulas and $F \rightarrow G$ is satisfiable, then F is not valid or G is satisfiable.

Part (b)

Refute the following statement: If F, G, and H are first-order formulas and $F \vee H \models G \vee H$, then $F \models G$.

Problem 2 *(Ordered resolution with selection)* (3 + 3 + 2 = 8 points)

Let $\Sigma = (\Omega, \Pi)$ be a signature with $\Omega = \{a/0,\ b/0,\ c/0,\ g/1,\ f/2\}$ and $\Pi = \{p/2,\ q/2,\ r/2\}$. Let N be the following set of clauses:

$$p(g(z), z) \qquad (1)$$

$$\boxed{\neg p(x, y)} \vee \neg p(x, a) \vee r(y, y) \qquad (2)$$

$$q(x, x) \qquad (3)$$

$$\neg q(b, b) \vee \boxed{\neg r(b, c)} \qquad (4)$$

$$\neg q(f(a, z), z) \vee \neg r(c, c) \qquad (5)$$

$$\neg q(g(a), g(y)) \vee \neg r(b, y) \qquad (6)$$

Suppose that an atom ordering $\succ$ is defined in such a way that $p(\ldots) \succ q(\ldots) \succ r(\ldots)$, and that the selection function S selects the framed literals.

Part (a)

If one saturates N with respect to the ordered resolution calculus with selection $Res_S^{\succ}$, then there is no inference between clauses (3) and (4) and no inference between clauses (3) and (5). Why? Explain briefly.

Part (b)

How many $Res_S^{\succ}$-inferences with premises in N are possible? What are their conclusions?

Part (c)

Saturate N with respect to $Res_S^{\succ}$. (Note: Compute only those inferences that are necessary according to the definition of $Res_S^{\succ}$ on slides 106 and 107.)

Problem 3 *(Prolog)* (7 points)

Let l be a list. A *splitting* of l is a list of lists $l' = [l_1, \ldots, l_n]$ with the following properties:

- none of the lists l_i is empty,
- the concatenation of all lists $l_1, \ldots, l_n$ equals l.

(For instance, the empty list `[]` has one splitting, namely `[]`, the list `[a]` has one splitting, namely `[[a]]`, the list `[a,b,c]` has four splittings, namely `[[a],[b],[c]]`, `[[a],[b,c]]`, `[[a,b],[c]]`, and `[[a,b,c]]`.) Implement a Prolog predicate `sp(`l`,`l'`)` that computes all splittings l' of a list l. You may use the predefined predicate `append`.

Problem 4 *(Logic programming)* (7 points)

Compute the canonical model of the following logic program:

$$p(f(f(a))).$$
$$p(b).$$
$$p(X) \leftarrow p(f(X)).$$

Problem 5 *(LTL)* (7 points)

Let $M = (S, x, L)$ be a linear time structure. Prove: $M, x \models \mathsf{F}p$ holds if and only if $M, x \models (\neg p)\,\mathsf{U}\,p$ holds.

Problem 6 *(CTL)* (7 points)

Give an example of a CTL state formula φ such that $M, s_0 \models \varphi$, but $M', s'_0 \not\models \varphi$.