Universität
des
Saarlandes
FR Informatik

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Tutorials for "Decision Procedures for Logical Theories"
Exercise sheet 9

Exercise 9.1: (*7 P.*)
Use the superposition calculus to refute the following set of clauses:

$$\begin{aligned}
&f(g(x)) \approx 0 \;\vee\; g(x) \approx b \\
&f(b) \approx 0 \\
\neg &f(g(a)) \approx 0 \;\vee\; g(a) \approx c \\
\neg &f(c) \approx 0
\end{aligned}$$

Use an ordering $\succ$ that compares two ground terms t and t' by lexicographically comparing first the number of f's in t and t', then the number of g's, then the number of a's, then the number of b's, and finally the number of c's. Use a selection function that selects all trivially false literals (that is, literals of the form $\neg t \approx t$) and nothing else. Perform only inferences that satisfy the conditions of the superposition calculus.

Exercise 9.2: (*5 P.*)
Let $\mathcal{A}$ and $\mathcal{A}'$ be Σ-algebras, let F be a Σ-formula. Prove: If $\mathcal{A}$ and $\mathcal{A}'$ are isomorphic, then $\mathcal{A} \models F$ if and only if $\mathcal{A}' \models F$.

Exercise 9.3: (*3 P.*)
Let $\mathcal{T} = (\mathbb{Q}, +)$. Use the Nelson/Oppen algorithm $\mathcal{NO}_{\mathcal{D}}[\mathcal{T}, \Phi]$ to check whether the constraint

$$\begin{aligned}
&2x + z \approx 0 \\
\wedge\; &2x' + z \approx 0 \\
\wedge\; &x + y' \approx y + x' + 1 \\
\wedge\; &f(x, x) \approx y \\
\wedge\; &f(x, x') \approx y'
\end{aligned}$$

over $\mathcal{T}^{\Phi}$ is satisfiable.

Exercise 9.4: (*5 P.*)
Prove Theorem 7.1: If all paths in a derivation tree from E_1, E_2 end in $\perp$, then E_1, E_2 is unsatisfiable in $\mathcal{T}_1 + \mathcal{T}_2$.

Put your solution into the mail box at the door of room 627 in the MPI building (46.1) before January 16, 14:00.