INFORMATIK

Universität des Saarlandes
FR Informatik

Harald Ganzinger
Uwe Waldmann

December 5, 2002

Tutorials for “Decision Procedures for Logical Theories”
Exercise sheet 7

Exercise 7.1: *(7 P.)*
Which of the following sentences are stable:

(a) $\exists x\,(-x^2 \geq 0)$

(b) $\exists x\,(-x^2 - 1 \geq 0)$

(c) $\exists x\,(-x^2 \geq 0 \ \vee\ x - 1 \geq 0)$

(d) $\forall x \exists y\,(xy^2 \geq 0)$

(e) $\exists x\,(25x^5 + x^2 - 1 \geq 0 \ \wedge\ 25x^5 + x^2 - 1 \leq 0)$

Exercise 7.2: *(7 P.)*
Which of the following laws hold (for variables x, y, z, and an arbitrary interval assignment ι)?

(a) $\mathcal{R}(\iota)(x + y) = \mathcal{R}(\iota)(y + x)$

(b) $\mathcal{R}(\iota)(x \cdot y) = \mathcal{R}(\iota)(y \cdot x)$

(c) $\mathcal{R}(\iota)((x + y) + z) = \mathcal{R}(\iota)(x + (y + z))$

(d) $\mathcal{R}(\iota)(x \cdot (y + z)) = \mathcal{R}(\iota)((x \cdot y) + (x \cdot z))$

(e) $\mathcal{R}(\iota)(\mathrm{sq}(x)) = \mathcal{R}(\iota)(x \cdot x)$, where sq is the squaring function over the reals.

For the laws that do not hold, find similar ones that do hold. Prove the resulting laws (use of properties presented in the lecture is allowed).

Exercise 7.3: *(6 P.)*
Which if the following properties of interval evalution of terms are still valid when doing it with (fixed-precision) floating point arithmetic with outward rounding?

(a) correctness

(b) convergence

(c) thinness

(d) optimality

Note: „outward rounding“ means that for every interval $[a, b]$, a is rounded to the largest machine-representable number that is smaller than or equal to a, and b is rounded to the smallest machine-representable number that is larger than or equal to b.

Put your solution into the mail box at the door of room 627 in the MPI building (46.1) before December 12, 14:00.