INFORMATIK

Universität des Saarlandes
FR Informatik

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Tutorials for "Decision Procedures for Logical Theories"
Exercise sheet 6

Exercise 6.1: (*7 P.*)
Do quantifier elimination in the theory of real closed fields on the following formulas.

(a) $\forall x\,(x^2 \geq y)$

(b) $\exists x \forall y\,(x + yz = 1)$

(c) $\exists x \forall y\,(y + xz = 1)$

(d) $\exists x\,(x^2 + xy + z < 0)$

(e) $\exists x\,(x^2 + y^2 \leq 1 \wedge x \geq y^2)$

Exercise 6.2: (*5 P.*)
Show that quantifier elimination for the the theory of real numbers with equality, addition, multiplication, and rational constants (but without inequality symbols) does not work: Give an example of a quantified formula F (with equality as the only predicate symbol) such that no quantifier-free formula *without* inequality symbols is equivalent to F, and give an equivalent quantifier-free formula *with* inequality symbols.

Exercise 6.3: (*3 P.*)
Define a predicate $\mathit{is\_a\_natural\_number}(x)$ using a first-order formula over the real numbers with equality, inequality, addition, multiplication, cosine, and rational constants.

Exercise 6.4: (*5 P.*)
Assume that we have a QE procedure for the theory of real closed fields. How can we do QE on a first-order formula that includes terms of the form $\sin(x)$ and $\cos(x)$, where the existentially quantified variable x does not occur anywhere else?

Put your solution into the mail box at the door of room 627 in the MPI building (46.1) before December 5, 14:00.