INFORMATIK

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FR Informatik

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Tutorials for "Decision Procedures for Logical Theories"
Exercise sheet 4

Exercise 4.1: (*5 P.*)
Use the congruence closure algorithm to prove or refute

$$\models_\Sigma \forall (g\,g\,x \approx g\,x \;\wedge\; f g\,g\,g\,x \approx g\,x \;\wedge\; f g\,x \approx f y \;\rightarrow\; f f f y \approx f y)$$

(To simplify the notation, we write fx instead of $f(x)$.)

Exercise 4.2: (*4 P.*)
Use the congruence closure algorithm to prove or refute

$$\models_\Sigma \forall (x + x \approx y + y \;\wedge\; (x + x) + x \approx x + x \;\rightarrow\; x + (y + y) \approx y + y)$$

(+ is a binary operator symbol written in infix notation.)

Exercise 4.3: (*5 P.*)
A binary relation $\rightarrow$ over a set S is called deterministic, if $s \rightarrow s_1$ and $s \rightarrow s_2$ implies $s_1 = s_2$ for all $s, s_1, s_2 \in S$. An element $s_0 \in S$ is called irreducible with respect to $\rightarrow$, if there is no $s \in S$ such that $s_0 \rightarrow s$. Prove: If $\rightarrow$ is deterministic and if s_0 is irreducible, then $s_1 \leftrightarrow^* s_0$ implies $s_1 \rightarrow^* s_0$.

Exercise 4.4: (*6 P.*)
Show how the halting problem for an arbitrary deterministic Turing machine with a single halting configuration can be reduced to the word problem for a particular equational theory. (Since there exist deterministic Turing machines with a single halting configuration for which the halting problem is undecidable, this implies that there exist equational theories for which the word problem is undecidable.) You can use Exercise 4.3.

Put your solution into the mail box at the door of room 627 in the MPI building (46.1) before November 21, 14:00.