Universität
des
Saarlandes
FR Informatik

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Tutorials for "Decision Procedures for Logical Theories"
Exercise sheet 3

Exercise 3.1: (*5 P.*)
A rewrite system R is called left-reduced, if for all rewrite rules $l \Rightarrow r \in R$, the term l is irreducible by $R \setminus \{l \Rightarrow r\}$. Show: If a ground rewrite system is terminating and left-reduced, then it is confluent.

Exercise 3.2: (*5 P.*)
Use Knuth-Bendix completion to transform the set of equations into an an equivalent canonical set of rewrite rules.

$$\begin{aligned} car(cons(x,y)) &\approx x \\ cdr(cons(x,y)) &\approx y \\ cons(car(x), cdr(x)) &\approx x \\ cdr(a) &\approx a \end{aligned}$$

Hint: Use the fact that a rewrite system is terminating, if for each rewrite rule the right-hand side is a proper subterm of the left-hand side.

Exercise 3.3: (*4 P.*)
Does the following rewrite system have critical pairs? Is it confluent?

$$\begin{aligned} \{\, f(x, g(x)) &\Rightarrow a, \\ f(x,x) &\Rightarrow b, \\ c &\Rightarrow g(c)\,\} \end{aligned}$$

Exercise 3.4: (*6 P.*)
Prove that the equational Horn clause

$$\forall (s_1 \approx t_1 \rightarrow s_0 \approx t_0)$$

over the signature Σ is universally valid, if and only if

$$s_1 \approx t_1 \models_{\Sigma(X)} s_0 \approx t_0$$

where variables in X are considered as additional constant symbols.

Put your solution into the mail box at the door of room 627 in the MPI building (46.1) before November 14, 14:00.

Note: In case of group work, write the names of all group members (not more than three!) on a single solution sheet. Do not submit several identical solution sheets.