Universität
des
Saarlandes
FR Informatik

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Tutorials for “Decision Procedures for Logical Theories”
Exercise sheet 11

Exercise 11.1: *(6 P.)*
Prove that the theory of the rational numbers with linear arithmetic is convex: Let x_j $(1 \le j \le n)$ be variables, let c_{ij} $(1 \le i \le m,\ 0 \le j \le n)$ be rational numbers, then

$$\mathbb{Q} \models \bigvee_{1 \le i \le m} \Big(\sum_{1 \le j \le n} c_{ij} x_j + c_{i0} = 0 \Big)$$

if and only if there exists some $i \in \{1, \dots, m\}$ with

$$\mathbb{Q} \models \sum_{1 \le j \le n} c_{ij} x_j + c_{i0} = 0 \,.$$

Note 1: $\mathbb{Q} \models \sum_{1 \le j \le n} c_{ij} x_j + c_{i0} = 0$ if and only if $c_{ij} = 0$ for all $j \in \{0, \dots, n\}$.

Note 2: Use an indirect proof and try to find sufficiently large values for the x_j so that the disjunction becomes false.

Note 3: Contrary to what was originally claimed in Example 7.9, there does not seem to exist an easy way to generalize the proof for two equations to an arbitrary number of equations or to reduce the case of an arbitrary number of equations to the special case of two equations.

Exercise 11.2: *(4 P.)*
Let $\mathcal{T} = (\mathbb{Q}, +)$. Use the Nelson/Oppen algorithm $\mathcal{NO}_{\mathcal{P}}[\mathcal{T}, \Phi]$ for convex theories to check whether the constraint

$$\begin{aligned}
& 2x + z \approx 0 \\
\wedge\ & 2x' + z \approx 0 \\
\wedge\ & x + y' \approx y + x' + 1 \\
\wedge\ & f(x, x) \approx y \\
\wedge\ & f(x, x') \approx y'
\end{aligned}$$

over $\mathcal{T}^{\Phi}$ is satisfiable.

Exercise 11.3: (*5 P.*)
Prove: If there exists a ground equation $s \approx t$ that holds in some structure of $\mathcal{T}$ and does not hold in some other structure of $\mathcal{T}$, then the theory $\mathcal{T}$ is not solvable.

Exercise 11.4: (*5 P.*)
Formulate a Shostak solver for the rational numbers with linear arithmetic using an appropriate inference system.

Put your solution into the mail box at the door of room 627 in the MPI building (46.1) before January 30, 14:00.