I N F O R M A T I K

Universität des Saarlandes
FR Informatik

Harald Ganzinger
Uwe Waldmann

January 16, 2003

Tutorials for "Decision Procedures for Logical Theories"
Exercise sheet 10

Exercise 10.1: *(6 P.)*
A theory $\mathcal{T}$ is called *closed under products*, if whenever $\mathcal{A}$ and $\mathcal{B}$ are models in $\mathcal{T}$ then also their product $\mathcal{A} \times \mathcal{B}$ is in $\mathcal{T}$. The product $\mathcal{A} \times \mathcal{B}$ has as domain the cartesian product of the domains of $\mathcal{A}$ and $\mathcal{B}$, and functions are defined componentwise as

$$f_{\mathcal{A}\times\mathcal{B}}((a_1, b_1), \dots, (a_n, b_n)) = (\,(f_{\mathcal{A}}(a_1, \dots, a_n), f_{\mathcal{B}}(b_1, \dots, b_n)\,)\,.$$

Prove: If $\mathcal{T}$ is closed under products, then $\mathcal{T}$ is convex.

Exercise 10.2: *(4 P.)*
Let $\mathcal{F}$ be a set of universally quantified equational Horn clauses (i. e., clauses that do not contain more than one positive literal). Prove: The class of all non-trivial algebras satisfying $\mathcal{F}$ is closed under products (and therefore convex).

Exercise 10.3: *(6 P.)*
Consider the theory of the rational numbers with addition *and multiplication* over the signature that contains all rational numbers as constant symbols and the binary operators $+$ (addition) and $\cdot$ (multiplication). Is it stably infinite? Is it convex? Prove or refute.

Exercise 10.4: *(6 P.)*
Let $\Sigma_1 = \{0/0, \mathit{succ}/1, -/1\}$, let $\mathcal{A}_1$ be the Σ_1-algebra whose domain is $\mathbb{Z}$ (with the standard interpretation of *succ* and unary $-$), and let $\mathcal{T}_1$ be the class of all Σ_1-algebras that are isomorphic to $\mathcal{A}_1$.

Let $\Sigma_2 = \{a/0, b/0, c/0, f/1, g/1\}$, let $\mathcal{A}_2$ be the term algebra over Σ_2, and let $\mathcal{T}_2$ be the class of all Σ_2-algebras that are isomorphic to $\mathcal{A}_2$.

Give a concrete example of an element of the amalgamation $\mathcal{T}_1 + \mathcal{T}_2$ (that is, describe its domain and the interpretations of the function symbols explicitly).

Put your solution into the mail box at the door of room 627 in the MPI building (46.1) before January 23, 14:00.