Universität des Saarlandes
FR Informatik

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Tutorials for "Decision Procedures for Logical Theories"
Exercise sheet 1

Exercise 1.1: *(5 P.)*
Prove that the canonical list model $\mathbb{L}$ described on slide 39 satisfies the list axioms. (Hint: Use the fact that L is uniquely normalizing.)

Exercise 1.2: *(5 P.)*
Find some non-trivial model of the list axioms that is not isomorphic to the canonical list model $\mathbb{L}$. (An algebra is trivial, if its universe is a one-element set.)

Exercise 1.3: *(6 P.)*
Let $\Sigma = (\Omega, \Pi)$ be a signature, and let $\mathcal{A} = (U_\mathcal{A}, (f_\mathcal{A} : U_\mathcal{A}^n \to U_\mathcal{A})_{f/n \in \Omega}, (p_\mathcal{A} \subseteq U_\mathcal{A}^m)_{p/m \in \Pi}))$ and $\mathcal{B} = (U_\mathcal{B}, (f_\mathcal{B} : U_\mathcal{B}^n \to U_\mathcal{B})_{f/n \in \Omega}, (p_\mathcal{B} \subseteq U_\mathcal{B}^m)_{p/m \in \Pi}))$ be two Σ-algebras. The product of $\mathcal{A}$ and $\mathcal{B}$ is a Σ-algebra

$$\mathcal{A} \times \mathcal{B} = (U_{\mathcal{A}\times\mathcal{B}}, (f_{\mathcal{A}\times\mathcal{B}} : U_{\mathcal{A}\times\mathcal{B}}^n \to U_{\mathcal{A}\times\mathcal{B}})_{f/n \in \Omega}, (p_{\mathcal{A}\times\mathcal{B}} \subseteq U_{\mathcal{A}\times\mathcal{B}}^m)_{p/m \in \Pi}))$$

defined as follows:

$$U_{\mathcal{A}\times\mathcal{B}} = U_\mathcal{A} \times U_\mathcal{B},$$
$$f_{\mathcal{A}\times\mathcal{B}}((a_1, b_1), \ldots, (a_n, b_n)) = (f_\mathcal{A}(a_1, \ldots, a_n), f_\mathcal{B}(b_1, \ldots, b_n)),$$
$$p_{\mathcal{A}\times\mathcal{B}} = p_\mathcal{A} \times p_\mathcal{B}.$$

Prove: If $\mathcal{F}$ is a set of universally quantified Σ-equations, and $\mathcal{A}$ and $\mathcal{B}$ are models of $\mathcal{F}$, then $\mathcal{A} \times \mathcal{B}$ is a model of $\mathcal{F}$. (In fact, the same property holds also for products of more than two, or even of infinitely many Σ-algebras.)

Exercise 1.4: *(4 P.)*
The property described in the previous exercise holds for sets of equations but not for sets of arbitrary formulas. Give an example of a set $\mathcal{F}$ of universally quantified formulas and two models of $\mathcal{F}$ such that their product is not a model of $\mathcal{F}$. (Hint: Find an equational formula F such that each model of F has at most two elements.)

Put your solution into the mail box at the door of room 627 in the MPI building (46.1) before October 31, 14:00.

Note: In case of group work, write the names of all group members (not more than three!) on a single solution sheet. Do not submit several identical solution sheets.