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Tutorials for "Automated Reasoning II"
Exercise sheet 4

Exercise 4.1:
Convexity of theories is sometimes defined in such a way that one considers only equations *between variables* in the right-hand side of the implication. Prove that the two definitions are equivalent: A first-order theory $\mathcal{T}$ is convex w.r.t. equations if and only if for every conjunction Γ of Σ-equations and non-equational literals and for all equations $x_i \approx x'_i$ $(1 \leq i \leq n)$, whenever $\mathcal{T} \models \forall\vec{x}\,(\Gamma \rightarrow x_1 \approx x'_1 \vee \cdots \vee x_n \approx x'_n)$, then there exists some index j such that $\mathcal{T} \models \forall\vec{x}\,(\Gamma \rightarrow x_j \approx x'_j)$.

Exercise 4.2:
Let $\Sigma = (\Omega, \emptyset)$ be a signature without predicate symbols (except built-in equality). For two Σ-algebras $\mathcal{A}$ and $\mathcal{B}$, we define the product $\mathcal{A} \times \mathcal{B}$ as the Σ-algebra whose universe is the cartesian product of the universes of $\mathcal{A}$ and $\mathcal{B}$, and where $f_{\mathcal{A}\times\mathcal{B}}((a_1, b_1), \ldots, (a_n, b_n)) = (\, f_{\mathcal{A}}(a_1, \ldots, a_n), f_{\mathcal{B}}(b_1, \ldots, b_n)\,)$.

A Σ-theory $\mathcal{T}$ is called closed under products, if the product of any two models of $\mathcal{T}$ is again a model of $\mathcal{T}$.

Prove: If $\mathcal{T}$ is closed under products, then it is convex.

Exercise 4.3:
Prove: If the axioms of the Σ-theory $\mathcal{T}$ are unversally quantified equational Horn clauses (that is, clauses where all atoms are equations and at most one of the literals is negated), then $\mathcal{T}$ is convex. (You may use the previous exercise.)

Bring your solution (or solution attempt) to the tutorial on Nov. 28.