



Uwe Waldmann

May 8, 2024

Tutorials for “Automated Reasoning II”
Exercise sheet 2

Exercise 2.1:

(1) Use the nondeterministic Nelson–Oppen method to show that the following formula is unsatisfiable in the combination of EUF and linear rational arithmetic:

$$\exists x, y (x + y \approx 0 \wedge f(x) - f(-y) \approx 1)$$

(If you choose the equations to split cleverly, the proof is quite short.)

(2) Use the deterministic Nelson–Oppen method for the same problem.

Exercise 2.2:

Find a simple example that demonstrates that the deterministic Nelson–Oppen combination procedure is incomplete if one of the theories is not convex.

Exercise 2.3:

Let $\Sigma = (\Omega, \emptyset)$ be a signature without predicate symbols (except built-in equality). For two Σ -algebras \mathcal{A} and \mathcal{B} , we define the product $\mathcal{A} \times \mathcal{B}$ as the Σ -algebra whose universe is the cartesian product of the universes of \mathcal{A} and \mathcal{B} , and where $f_{\mathcal{A} \times \mathcal{B}}((a_1, b_1), \dots, (a_n, b_n)) = (f_{\mathcal{A}}(a_1, \dots, a_n), f_{\mathcal{B}}(b_1, \dots, b_n))$.

A Σ -theory \mathcal{T} is called closed under products, if the product of any two models of \mathcal{T} is again a model of \mathcal{T} .

Prove: If \mathcal{T} is closed under products, then it is convex.

Exercise 2.4:

Prove: If the axioms of the Σ -theory \mathcal{T} are universally quantified equational Horn clauses (that is, clauses where all atoms are equations and at most one of the literals is positive), then \mathcal{T} is convex. (You may use the previous exercise.)

Exercise 2.5:

Is the theory of abelian groups stably infinite? Give an explanation. (Hint: If $(G_1, +_1, 0_1)$ and $(G_2, +_2, 0_2)$ are abelian groups, then the cartesian product $(G_1 \times G_2, +, 0)$ with $(x, y) + (x', y') := (x +_1 x', y +_2 y')$ and $0 := (0_1, 0_2)$ is again an abelian group.)

Exercise 2.6:

Show that the theory described by the following set of axioms is not stably infinite.

$$\begin{aligned}\forall x (x * 0 \approx 0) \\ \forall x (x * 1 \approx x)\end{aligned}$$

Bring your solution (or solution attempt) to the tutorial on May 13.