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Tutorials for “Automated Reasoning II”
Exercise sheet 6

Exercise 6.1:

A group is a set G with a binary function $\cdot : G \times G \rightarrow G$, a unary function ${}^{-1} : G \rightarrow G$, and an element $e \in G$ that satisfy the axioms

$$\begin{aligned}a \cdot (b \cdot c) &= (a \cdot b) \cdot c \\a \cdot e &= a \\a \cdot a^{-1} &= e\end{aligned}$$

for all $a, b, c \in G$. (It is sufficient to assert that e is a right identity and that ${}^{-1}$ is a right inverse. One can prove from these axioms that e is also a left identity and that ${}^{-1}$ is also a left inverse.)

For a group element $a \in G$ and a positive integer n , we define a^n recursively by $a^1 = a$ and $a^{n+1} = a \cdot (a^n)$. We say that $a \in G$ has order n if n is the smallest positive integer such that $a^n = e$. We say that $a \in G$ has order ∞ if there is no positive integer n such that $a^n = e$. (Note that every group has exactly one element with order 1, namely e itself.)

We say that some $a \in G$ commutes with some $b \in G$ if $a \cdot b = b \cdot a$. The center of a group G is the set of all elements $a \in G$ that commute with every $b \in G$.

Formalize the following problem in unsorted first-order logic with equality and use the theorem prover SPASS to prove it: If a group G has exactly one element with order 2, then this element is in the center of G .

Notes:

- You can download SPASS 3.9 from <http://www.spass-prover.org/> or use the web interface.
- A sample SPASS input file containing the definition of a group and the conjecture that the right identity element in a group is also a left identity is available from the tutorial web page.
- Even though the presentation above refers to integer numbers, you should formalize the problem without referring to integer numbers.
- It is advisable to formalize the problem without defining auxiliary predicates like `commutes(_)` or `center(_)`. (With auxiliary predicates, the problem becomes noticeably harder for SPASS.)

Exercise 6.2:

Find an unsatisfiable clause set consisting of two unit clauses $s \approx t$ and $u \not\approx v$ and a term ordering \succ such that the only inference that neither violates the ordering restrictions of the superposition calculus nor yields a tautology is a positive superposition inference in which the left-hand side of $s \approx t$ is unified with the right-hand side of a renamed copy of $s \approx t$.

Exercise 6.3:

Let N be the set of constrained clauses

$$f(g(x)) \approx x \llbracket \top \rrbracket \quad (1)$$

$$h(b) \approx c \llbracket \top \rrbracket \quad (2)$$

Are the following clauses redundant w. r. t. N , if we define redundancy as on page 66 of the lecture notes?

$$f(h(x)) \approx f(c) \llbracket x = b \rrbracket \quad (3)$$

$$h(f(g(b))) \approx c \llbracket \top \rrbracket \quad (4)$$

$$h(f(g(x))) \approx h(x) \llbracket \top \rrbracket \quad (5)$$

Submit your solution (or solution attempt) by e-mail to uwe@mpi-inf.mpg.de, subject Ex 6. until July 12.