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Tutorials for "Automated Reasoning II"
Exercise sheet 4

Exercise 4.1:
Let $\Sigma = (\Omega, \emptyset)$ be a signature without predicate symbols (except built-in equality). For two Σ-algebras $\mathcal{A}$ and $\mathcal{B}$, we define the product $\mathcal{A} \times \mathcal{B}$ as the Σ-algebra whose universe is the cartesian product of the universes of $\mathcal{A}$ and $\mathcal{B}$, and where $f_{\mathcal{A}\times\mathcal{B}}((a_1, b_1), \ldots, (a_n, b_n)) = \big(f_{\mathcal{A}}(a_1, \ldots, a_n), f_{\mathcal{B}}(b_1, \ldots, b_n)\big)$.

A Σ-theory $\mathcal{T}$ is called closed under products, if the product of any two models of $\mathcal{T}$ is again a model of $\mathcal{T}$.

Prove: If $\mathcal{T}$ is closed under products, then it is convex.

Exercise 4.2:
Prove: If the axioms of the Σ-theory $\mathcal{T}$ are unversally quantified equational Horn clauses (that is, clauses where all atoms are equations and at most one of the literals is positive), then $\mathcal{T}$ is convex. (You may use the previous exercise.)

Exercise 4.3:
In many applications of CDCL or CDCL(T), one does not only want a yes/no answer, but also an explanation for it. In the case of an unsatisfiable input, this explanation is an "unsatisfiable core", i.e., a (small) subset of the input clauses that is already sufficient to show $\mathcal{T}$-inconsistency. How can we get an unsatisfiable core from a CDCL(T) proof?

Exercise 4.4:
Refute the following set of equational clauses by superposition:

$$f(x) \not\approx a \;\vee\; f(x) \approx b \quad (1)$$
$$f(f(x)) \approx x \quad (2)$$
$$a \not\approx b \quad (3)$$

Choose an appropriate ordering and perform only inferences that satisfy the ordering restrictions.

Submit your solution (or solution attempt) by e-mail to `uwe@mpi-inf.mpg.de`, subject `Ex 4`. until June 16.