max planck institut
informatik

Universität
des
Saarlandes
FR Informatik

Uwe Waldmann **May 18, 2020**

Tutorials for "Automated Reasoning II"
Exercise sheet 2

Exercise 2.1:
The quantifier elimination algorithms for linear rational arithmetic can also be applied to non-linear formulas, provided that all the bound variables occur only linearly. That is, the atoms can have the form $\sum_i s_i(\vec{z}) \cdot x_i \sim 0$ where the coefficients $s_i(\vec{z})$ are terms that may contain arbitrary arithmetic operations, say $(z_1 + z_3^2)$ or even $(\sin z_2 + e^{z_5} + 3)$, but no bound variables. There is one additional problem, though. Why? How can you solve it?

Exercise 2.2:
Use the nondeterministic Nelson–Oppen method to show that the following formula is unsatisfiable in the combination of EUF and linear integer arithmetic:

$$\exists x, y\, (x + y \approx 0 \;\wedge\; f(x) + f(-y) \approx 1)$$

(If you choose the equations to split cleverly, the proof is quite short.)

Exercise 2.3:
Read the proof of Lemma 1.7 in the lecture notes. Explain the following step:

> Since the equations $x \approx y$, with $x \sim y$, are entailed by F and since F is satisfiable, this means that this equation must come from the last disjunct.

Exercise 2.4:
The conditions of Thm. 1.8 are a bit stronger than necessary. Can you think about weaker conditions that are still sufficient to prove the theorem (with almost the same proof)?

Exercise 2.5:
Show that the theory described by the following set of axioms is not stably infinite.

$$\forall x\, (x * 0 \approx 0)$$
$$\forall x\, (x * 1 \approx x)$$

Submit your solution (or solution attempt) by e-mail to `uwe@mpi-inf.mpg.de`, subject `Ex 2`. until May 24.