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May 23, 2018

Tutorials for "Automated Reasoning II"
Exercise sheet 6

Exercise 6.1:
A group is a set G with a binary function $\cdot : G \times G \to G$, a unary function $\_^{-1} : G \to G$, and an element $e \in G$ that satisfy the axioms

$$a \cdot (b \cdot c) = (a \cdot b) \cdot c$$
$$a \cdot e = a$$
$$a \cdot a^{-1} = e$$

for all $a, b, c \in G$. (It is sufficient to assert that e is a right identity and that $\_^{-1}$ is a right inverse. One can prove from these axioms that e is also a left identity and that $\_^{-1}$ is also a left inverse.)

For a group element $a \in G$ and a positive integer n, we define a^n recursively by $a^1 = a$ and $a^{n+1} = a \cdot (a^n)$. We say that $a \in G$ has order n if n is the smallest positive integer such that $a^n = e$. We say that $a \in G$ has order ∞ if there is no positive integer n such that $a^n = e$. (Note that every group has exactly one element with order 1, namely e itself.)

We say that some $a \in G$ commutes with some $b \in G$ if $a \cdot b = b \cdot a$. The center of a group G is the set of all elements $a \in G$ that commute with every $b \in G$.

Formalize the following problem in unsorted first-order logic with equality and use the theorem prover SPASS to prove it: If a group G has exactly one element with order 2, then this element is in the center of G.

Notes:

- You can download SPASS 3.9 from `http://www.spass-prover.org/` or use the web interface.
- A sample SPASS input file containing the definition of a group and the conjecture that the right identity element in a group is also a left identity is available from the tutorial web page.
- Even though the presentation above refers to integer numbers, you should formalize the problem without referring to integer numbers.

- It is advisable to formalize the problem without defining auxiliary predicates like `commutes(_)` or `center(_)`. (With auxiliary predicates, the problem becomes noticably harder for SPASS.)

Exercise 6.2:
Find an unsatisfiable clause set consisting of two unit clauses $s \approx t$ and $u \not\approx v$ and a term ordering $\succ$ such that the only inference that neither violates the ordering restrictions of the superposition calculus nor yields a tautology is a positive superposition inference in which the left-hand side of $s \approx t$ is unified with the right-hand side of a renamed copy of $s \approx t$.

Exercise 6.3:
Let N be a set of equational clauses such that $\bot \notin N$. In Thm. 3.9, we have shown that whenever N is saturated up to redundancy, then every ground instance $C\theta \in G_\Sigma(N)$ is either productive or true in $R_{C\theta}$. The converse does not hold, not even for ground unit clauses: Give a (small) set of ground unit clauses N such that $\bot \notin N$ and every $C \in N$ is either productive or true in R_C, but N is not saturated up to redundancy.

Exercise 6.4:
How would you redefine the fairness of a run if saturation is defined using redundant inferences? Try to find the easiest possible definition. Reprove Lemma 3.16 for the new definitions of saturation and fairness.

Bring your solution (or solution attempt) to the tutorial on May 30.