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Tutorials for “Automated Reasoning II”
Exercise sheet 5

Exercise 5.1:
Prove that the multiset extension of a reduction ordering is stable under substitutions (which implies that the literal ordering defined on page 50 of the lecture notes is stable under substitutions). Note: There are several ways to characterize a multiset ordering, see e.g. the lecture notes from the previous semester or the book by Baader and Nipkow. You may pick the most convenient one for this purpose.

Exercise 5.2:
On page 50 of the lecture notes it is stated that the ordering restrictions of the inference rules of the superposition calculus must be satisfied *after applying the mgu to the premises.* Give a simple example that shows that a literal may be maximal in a clause, but that the maximality requirement may be violated after applying the mgu.

Exercise 5.3:
Let $D = D' \vee t \approx t'$ and $C[u]$ be two clauses such that there is a (positive or negative) superposition inference between D and C with conclusion $C_0 = (D' \vee C[t'])\sigma$, where σ is the mgu of t and u. Suppose that $t\sigma$ occurs at least once in $C[t']\sigma$. Let C_0' be the clause that we obtain from C_0 if every occurrence of $t\sigma$ within $C[t']\sigma$ is replaced by $t'\sigma$. (As an example, consider $D = g(x) \not\approx g(y) \vee f(x,y) \approx f(y,x)$, $C = h(f(g(b),z)) \approx f(g(b),z)$, $t = f(x,y)$, $t\sigma = f(g(b),z)$, $C_0 = g(g(b)) \not\approx g(z) \vee h(f(z,g(b))) \approx f(g(b),z)$, $C_0' = g(g(b)) \not\approx g(z) \vee h(f(z,g(b))) \approx f(z,g(b))$.)

(a) C_0' is entailed by D and C_0. Why?

(b) C_0 is not redundant w. r. t. $\{D, C_0'\}$. Why?

(c) The inference that produces C_0 is redundant w. r. t. $\{D, C_0'\}$. Why?

Hint 1: Read the definitions of redundant inferences and instances of inferences really carefully. Hint 2: The ordering restrictions are an integral part of the definition of superposition inferences.

Exercise 5.4:
Compute R_∞ for the clause set $\{f(x) \approx a\}$ and the signature $\Sigma = (\{f/1,\ g/1,\ a/0\}, \emptyset)$; use the LPO with $g > f > a$.

Bring your solution (or solution attempt) to the tutorial on May 22.