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Tutorials for "Automated Reasoning II"
Exercise sheet 4

Exercise 4.1:
In many applications of CDCL or CDCL(T), one does not only want a yes/no answer, but also an explanation for it. In the case of an unsatisfiable input, this explanation is an "unsatisfiable core", i.e., a (small) subset of the input clauses that is already sufficient to show $\mathcal{T}$-inconsistency. How can we get an unsatisfiable core from a CDCL(T) proof?

Exercise 4.2:
Many decision procedures detect the unsatisfiability of a set of literals by iteratively deriving new literals from given literals; if an inconsistent literal is derived at the end, the input is unsatisfiable. Examples include Gaussian elimination or the Fourier-Motzkin procedure.

For such decision procedures, it is easy to generate explanations for unsatisfiability. We associate a set $E(L)$ of input literals to each literal L: for input literals L, $E(L) := \{L\}$; for literals L derived from ancestor literals $L_1, \ldots, L_n$, $E(L) := E(L_1) \cup \cdots \cup E(L_n)$. When an inconsistent literal L_0 is derived at the end, $E(L_0)$ yields the explanation.

However, the explanations computed in this way are not always minimal. Consider the following set of equations in linear rational arithmetic:

$$\begin{aligned} x - 2z &= 1 && (1)\\ -x + y - 3w &= 3 && (2)\\ z - 2w &= 0 && (3)\\ 2x - 2y + 3z &= 5 && (4) \end{aligned}$$

If we use equation (1) to eliminate x from the other equations, then (2) to eliminate y, then (3) to eliminate z, equation (4) is turned into $0 = 11$. All four equations were involved in this derivation; still $\{(1), (2), (3), (4)\}$ is *not* a minimal explanation for the contradiction. How could one efficiently find a smaller explanation? (Hint: Think about linear combinations.)

Exercise 4.3:
Prove that the Equality Factoring rule (page 49 of the lecture notes) is sound.

Exercise 4.4:
Refute the following set of equational clauses by superposition:

$$\begin{aligned} f(x) \not\approx a \vee f(x) \approx b \quad & (1) \\ f(f(x)) \approx x \quad & (2) \\ a \not\approx b \quad & (3) \end{aligned}$$

Choose an appropriate ordering and perform only inferences that satisfy the ordering restrictions.

Bring your solution (or solution attempt) to the tutorial on May 16.