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Tutorials for “Automated Reasoning II”
Exercise sheet 1

Exercise 1.1:
Use the congruence closure algorithm to check whether the equational clause

$$\forall x, y\ f(f(x)) \not\approx x \vee f(x) \not\approx y \vee f(f(y)) \not\approx g(y) \vee x \approx y \vee h(x, y) \approx h(x, g(y))$$

is valid.

Exercise 1.2:
Let $\Sigma = (\Omega, \Pi)$ and $\Sigma' = (\Omega', \Pi')$ be signatures such that $\Omega \subseteq \Omega'$ and $\Pi \subseteq \Pi'$. If $\mathcal{A}$ is a Σ-algebra and $\mathcal{B}$ is a Σ'-algebra such that $U_{\mathcal{A}} = U_{\mathcal{B}}$, $f_{\mathcal{A}} = f_{\mathcal{B}}$ for every $f \in \Omega$ and $P_{\mathcal{A}} = P_{\mathcal{B}}$ for every $P \in \Pi$, then $\mathcal{A}$ is called the Σ-reduct of $\mathcal{B}$ (denoted by $\mathcal{A} = \mathcal{B}|_{\Sigma}$).

Prove the following two properties:

(i) Let $\Sigma = (\Omega, \Pi)$ with $c \notin \Omega$, let $\Sigma' = (\Omega \cup \{c/0\}, \Pi)$. Let N be a set of closed Σ-formulas and let t be a ground term occurring in a closed Σ-formula $F[t]_p$. Then the Σ-models of $N \cup \{F[t]_p\}$ are exactly the Σ-reducts of the Σ'-models of $N \cup \{F[c]_p\} \cup \{t \approx c\}$.

(ii) Let N be a set of closed Σ-formulas, let N' be a set of closed Σ'-formulas, and let F be a closed Σ-formula. If the Σ-models of N are exactly the Σ-reducts of Σ'-models of N', then $N \models F$ if and only if $N' \models F$.

Exercise 1.3:
The Fourier-Motzkin algorithm would be unsound if we omitted the non-triviality axioms from the definition of ODAGs. Where do we need non-triviality?

Exercise 1.4:
Use the Loos-Weispfenning algorithm to eliminate $\exists x$ from the formula

$$\exists x\ \big((2x - y > 0 \vee x \geq 2) \wedge (y - x \geq -1 \vee x < 1) \wedge x < 3y\big)$$

Simplify the result as much as possible.

Bring your solution (or solution attempt) to the tutorial on April 18.