Assignment 1 *(CDCL)* (14 points)

Let N be the following set of propositional clauses:

$$
\begin{array}{rl}
(1) & \neg P \lor T \lor U \\
(2) & \neg P \lor T \lor \neg V \\
(3) & P \lor \neg Q \\
(4) & P \lor Q \lor \neg R \lor T \lor \neg U \lor V \\
(5) & \neg R \lor \neg S \\
(6) & R \lor \neg S \lor T \\
(7) & S \lor T \\
(8) & P \lor \neg T \lor U \\
(9) & \neg T \lor V \\
(10) & \neg T \lor \neg U \lor \neg V
\end{array}
$$

Use the CDCL procedure to check whether N is satisfiable or not; if it is satisfiable, give a total model. Use a reasonable strategy. If you use the *Decide* rule, use the largest undefined negative literal according to the ordering $\neg P > \neg Q > \neg R > \neg S > \neg T > \neg U > \neg V$. If you use the *Backjump* rule, determine a suitable backjump clause using the 1UIP method and use the best possible successor state for that backjump clause.

Assignment 2 *(Semantics)* (12 points)

Let Π be a set of propositional variables; let Q and R be two propositional variables in Π. For any Π-formula F let $\mathrm{rep}(F)$ be the formula that one obtains by replacing every occurrence of Q in F by R.

Prove: If $\mathrm{rep}(F)$ is satisfiable, then F is satisfiable. (It is sufficient if you consider propositional variables, conjunctions $G \land G'$, and negations $\neg G$; the other cases are handled analogously.)

Assignment 3 *(Tseitin transformation)* (6 + 6 = 12 points)

Let Π be a set of propositional variables. Let $Q \in \Pi$. Let $H[F]_p$ be a valid Π-formula in which Q does not occur.

Part (a) What can we say about the satisfiability of $H[Q]_p \land (Q \leftrightarrow F)$? Is it (1) necessarily satisfiable, (2) possibly but not necessarily satisfiable, or (3) necessarily unsatisfiable? Explain briefly.

Part (b) What can we say about the validity of $H[Q]_p \land (Q \leftrightarrow F)$? Is it (1) necessarily valid, (2) possibly but not necessarily valid, or (3) necessarily non-valid? Explain briefly.

Assignment 4 *(CNF transformation)* (12 points)

Let F be the propositional formula $((P \leftrightarrow \neg Q) \wedge R) \rightarrow (\neg P \wedge Q)$. Use the optimized CNF transformation (lecture notes, page 26/27) to convert F into a CNF formula. In step 2, replace the subformula $(P \leftrightarrow \neg Q)$ by a new variable.

Assignment 5 *(Propositional logic)* (12 points)

Let F, G, H be formulas; let P, Q be propositional variables; let C, D be clauses; let N be a set of clauses; and let $\mathcal{A}$ be a valuation. Decide for each of the following statements whether it is true or false:

(1) If F is satisfiable and G is unsatisfiable, then $F \wedge \neg G$ is satisfiable.

(2) If G is satisfiable and $F \models G$, then F is satisfiable.

(3) If $\mathrm{pol}(H, p) = 1$, then $H[F \wedge G]_p \models H[F]_p$.

(4) $\mathcal{A} \models G \vee H$ if and only if $\mathcal{A} \models G$ or $\mathcal{A} \models H$.

(5) $F \models G \vee H$ if and only if $F \models G$ or $F \models H$.

(6) If N contains C, then $N \models (C \vee D)$.

(7) If N contains $P \vee Q \vee C$ and $\neg P \vee \neg Q \vee D$, then N and $N \cup \{C \vee D\}$ are equisatisfiable.

(Note on grading: A yes/no answer is sufficient; you do not have to give an explanation. However, you need at least four correct answers to get any points for this assignment. Missing answers count like false answers.)

Assignment 6 *(Orderings, Termination)* (6 + 6 + 6 = 18 points)

Part (a) Let $M = \{a, b, c, d\}$. Suppose that the binary relation $\rightarrow$ over multisets over M is defined by the rules (1)–(3):

$$
\begin{array}{lrcl}
(1) & S \cup \{b, c\} & \rightarrow & S \cup \{a, a, a\} \\
(2) & S \cup \{b, a\} & \rightarrow & S \cup \{b, c, c\} \\
(3) & S \cup \{c\} & \rightarrow & S \cup \{d\}
\end{array}
$$

Then $\rightarrow$ can be shown to be terminating using the multiset extension $\succ_{\mathrm{mul}}$ of an appropriate well-founded ordering on M. How does $\succ$ look like?

Part (b) If the binary relation $\rightarrow$ is defined by the rules (4)–(6),

$$
\begin{array}{lrcl}
(4) & S \cup \{a, a\} & \rightarrow & S \cup \{b, c\} \\
(5) & S \cup \{b, b\} & \rightarrow & S \cup \{a, c\} \\
(6) & S \cup \{b, c\} & \rightarrow & S \cup \{a, d, c, c\}
\end{array}
$$

then there is no well-founded ordering on M such that $\rightarrow$ is contained in $\succ_{\mathrm{mul}}$. Why? Give a short explanation.

Part (c) Still the relation $\rightarrow$ defined by the rules (4)–(6) is terminating. Prove it. (Hint: Think about lexicographic combinations.)